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A Planar Hopping Robot with One Actuator:

Design, Simulation, and Experimental Results

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Abstract — **Animal legs are much more complex than necessary for running on horizontal surfaces. Past research shows that, despite their complexity, they act essentially like simple pogo sticks, or the "Spring Loaded Inverted Pendulum" (SLIP). In this paper, the possibility of the realization of a SLIP model using a spring in the leg and only one actuator at the hip is investigated first in numerical simulation and then in experiments. A simple controller results in running at 0.80 *m/s* (6.7 leg lengths per second). The stability and robustness of the resulting motion is demonstrated.**

Keywords – SLIP; hopping robot; one-legged; unactuated

I. INTRODUCTION

A. Motivation

If we first look to nature, animals can run with great speed and maneuverability even on rough terrain. For instance, cheetahs can gallop at 31 *m/s* and red kangaroos can leap at 16 *m/s*. Hence, it is a good idea to draw insight from animal locomotion. Kinematics, actuation, and control of animals are redundant [1]. In fact, biomechanics researchers have been trying to understand the running motion of animals through the use of simplified models [2] and robotics researchers have been exploiting these models for designing and controlling robots [3]. For three decades, the running gait has been described using the Spring Loaded Inverted Pendulum (SLIP) [1], [2]. The SLIP model consists of a point-mass on a springy leg. It represents the dynamics of the center of mass (COM) of a running animal and of the conceptual leg from the COM to the foot point on the ground, as in Fig. 1. The SLIP model motivated the design of the robot used in this research.

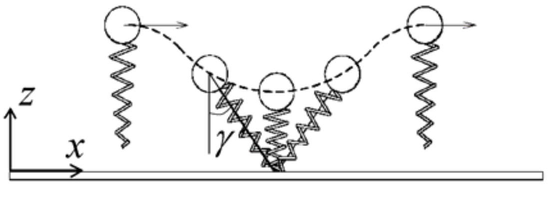

Figure 1. Spring Loaded Inverted Pendulum (SLIP) model

B. Goal

The SLIP model was adopted as a basis of the robot design and the goals of this research were set as follows:

- Feasibility study of the realization of the SLIP model
- Design and construction of a SLIP-based hopping robot with one actuator, named the SLIP Hopper
- Experimental implementation of a simple controller on the SLIP Hopper.

As a result, we achieved the following:

- Experimental validation of the SLIP model
- Demonstration of a robust hopping robot with only one actuator at the hip of the leg.

C. Related work and studied features of the SLIP model

The SLIP model has been used for 20 years among robotics researchers [3], [4], [5]. Raibert and his co-workers established the first milestone by using the SLIP as a basic dynamics model for his hopping robots [3]. Since then, many running robots have used Raibert's controller based on the SLIP model by applying it to their robots with complex kinematics. To date, the Bow Leg hopper [5] is one of the closest in its design to the SLIP model. It has an actuated compliant leg (the leg is pre-loaded during flight) consisting of a curved (bow shaped) fiberglass sheet. Its COM is located below the hip joint to allow the body to be largely self-stabilizing.

The SLIP is the minimum model for running on level surfaces [1], [2]. It has two DOF of translation in the sagittal plane as in Fig. 1. Since the body is represented by a point mass at the COM, its inertia and pitching motion are not considered. The SLIP has only one actuated joint at the hip, which is coincident with its COM of the body. The conceptual compliant leg can be regarded as an unactuated prismatic joint with a spring while the leg is in contact with the ground.

The dynamics of the ideal SLIP model is energetically conservative and naturally stable in the sense of its balance and does not require active balancing control provided that the leg can be brought at a proper angle at touchdown [6]. Due to this "self-stability" property, the control of the robot designed after the SLIP model can be simplified.

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Our robot is designed based on these features and takes advantage of expertise in the design of other legged robots [4], emphasizing simple and efficient sets of mechanical design and control schemes using the SLIP model.

II. DESIGN

Our robot, the SLIP Hopper, is designed as a realization of the SLIP model. In order to do numerical simulation for a feasibility study of its realization, the mechanical parameters of the SLIP are determined and a simple controller is prepared. After the simulation confirms the possibility, those parameters and controller are implemented on an actual experimental setup.

A. Mechanical Parameters

The SLIP model consists of a point-mass with a springy leg. Therefore, there are three mechanical parameters to be determined in order to build a SLIP robot: body mass, spring stiffness, and leg length.

1) Body Mass m

First, the mass value for the body is determined. The main components of the actual robot are a hip servo, leg, and mechanical interconnects. The total mass of the components is 0.32 *kg*. An additional aluminum block is attached for the future implementation of a motor amplifier and an embedded computer on the robot. It weighs 0.22 *kg*. The total mass of the body *m* is 0.54 *kg*.

2) Spring Stiffness k

The spring stiffness is designed for the dynamics in which the robot is on the ground, because when the leg is on the ground, the leg spring is used and its deflection is changed. The SLIP model can be regarded as a harmonic system with the leg's toe attached with a pin joint on the ground. Its natural frequency can be approximated by the equivalent harmonic system with vertical motion [3]. By setting the natural frequency f_n during stance phase at 3 *Hz*, we obtain $k = 193$ *N/m* using the following equations:

$$f_n = \frac{\omega_n}{2\pi} \quad \text{and} \quad \omega_n = \sqrt{\frac{k}{m}} \quad \rightarrow \quad k = (2\pi f_n)^2 m \qquad (1)$$

where ω_n is the natural angular frequency. The natural frequency of 2-3 Hz is a common value of stride frequencies found in nature [7] and in several legged robots in the Ambulatory Robotics Laboratory at McGill University [4].

3) Leg Length r

The rest leg length is set at 0.120 *m* so that the leg has the necessary prismatic joint travel for a desired hopping height of 0.170 *m*.

B. Controller

1) Phase Detection

Legged robots are event-driven intermittent dynamical systems. They change their dynamical characteristics depending on their leg conditions. Those conditions are called "phases" [3]. In the case of one-legged robots, there are only two phases. The leg is on the ground or it is not. If the leg is on the ground, it is called "stance phase". If it is not, it is called "flight phase". The phase transitions are driven by two events: touchdown and lift-off.

The robot needs a different associated control scheme for each phase because the dynamic model in each phase is different. Reflex control is used so that the entire locomotion controller can switch control methods at the appropriate moment. Therefore, the robot is required to detect the phase change at touchdown onto the ground and at lift-off from the ground. In simulation, the leg toe height is measured for that purpose, and in experiment, an infrared distance sensor is attached on the leg toe.

2) Leg Control

The robot now knows its phase and controls the leg depending on the phase. We trust the self-stability property of the SLIP [6] and the robot does not make any special effort for stabilization of hopping but simply sets a leg angle at touchdown and another at lift-off. In the flight phase, the leg is swung forwards to bring itself to the desired touchdown angle which will ensure gait stability. In the stance phase, the leg is swept backwards, which adds energy compensating for losses during the cycle of steady state running. The following PD control is used to achieve desired touchdown and lift-off angles of the leg:

$$\tau_m = K_P(\gamma_{desired} - \gamma_{actual}) + K_D(\dot{\gamma}_{desired} - \dot{\gamma}_{actual}) \qquad (2)$$

where τ_m is motor torque, K_P and K_D are gains, and γ is leg angle. The PD gains used in simulation and experiment are shown in TABLE II.

The desired forward speed is set to 0.80 *m/s*. In order to achieve this forward speed, the robot has to sweep the leg in such a way that the COM of the robot is propelled at this speed in the stance phase. In the flight phase, the COM is supposed to move forwards at the same speed as at lift-off. For a forward speed of 0.80 *m*/s, the range of angle between touchdown leg angle and lift-off leg angle is set 65 *deg*. This is the value by which the leg is swept in the stance phase and is obtained using Farley's *et al.* approximate geometry formula by assuming the leg length is constant throughout the stance phase [7].

The desired leg angles at touchdown and lift-off are 40 *deg* and –25 *deg*, respectively. These values are adjusted empirically while keeping the sweeping range to 65 *deg*.

TABLE I. PD GAINS

	Phases	**Gains**	**Values**	**Units**
Simulation	Flight	P	1.20×10^{-3}	*N m/deg*
		D	3.00×10^{-5}	*N m/s deg*
	Stance	P	1.40×10^{-3}	*N m/deg*
		D	2.20×10^{-5}	*N m/s deg*
Experiment	Flight	P	8.456×10^{-2}	*N m/deg*
		D	5.315×10^{-3}	*N m/s deg*
	Stance	P	7.248×10^{-2}	*N m/deg*
		D	7.248×10^{-3}	*N m /s deg*

III. SIMULATION IMPLEMENTATION

The robot is an intermittent and nonlinear dynamical system, and therefore it is often the case that, for analysis, analytical solutions are difficult to obtain and numerical simulation is the appropriate means for finding a solution. Numerical simulation is done in order to investigate the possibility of the realization of a SLIP and its stable hopping.

We have the mechanical parameters for a SLIP and a controller for it, and therefore, we have the necessary information for simulation. They are implemented in simulation using the Matlab and Simulink software packages.

A. *Dynamics Models*

Approximate models for the stance and flight phases are used while the essential characteristics of the SLIP model are preserved: a ballistic flight model and a planar harmonic stance model.

In the flight phase, it is assumed that the COM of the body flies along a ballistic trajectory since the COM is a point mass and the mass of the leg is included in the mass of the body. In the flight phase, however, the inertia of the leg is still taken into account in order to calculate the effect of hip torque on leg motion. The motion of the COM and that of the leg are considered to be decoupled.

In the stance phase, the SLIP is represented as a harmonic system in the sagittal plane. The inertia of the leg is neglected. Ground contact between the toe and the ground is modeled as a pin joint so that the toe does not slip on the ground surface. Friction of the ground contact is neglected.

The effect of impact at touchdown and lift-off is neglected as well though there is impact between the toe and the ground and there is impact between the prismatic joint slider and the mechanical stop of the prismatic joint on the actual robot, which is mentioned later.

The equations of motion are obtained for each phase model using the Lagrangian equation. For the flight phase,

$$\ddot{z} = -g, \quad \ddot{x} = 0, \quad \ddot{\gamma} = -\frac{\tau_h}{J} \tag{3}$$

and for the stance phase,

$$\begin{aligned} \ddot{r} &= r\dot{\gamma}^2 - g\cos\gamma + \frac{k(r_{rest} + \delta - r)}{m} - b\dot{r} \\ \ddot{\gamma} &= -\frac{2\dot{r}\dot{\gamma}}{r} + \frac{g\sin\gamma}{r} - \frac{\tau_h}{mr^2} \end{aligned} \tag{4}$$

where z is vertical position, x is horizontal position, γ is leg angle with respect to the vertical line, τ_h is hip torque, J is leg inertia, r is leg length, δ is pre-deflection of the spring, , and b is leg damping constant. The dry friction and viscous damping at the hip joint and in the prismatic leg are modeled by $b\dot{r}$. The parameter values in simulation including motor and gear parameter values are as in TABLE II.

TABLE II. PARAMETERS FOR DYNAMICS MODELS

Parameters	Symbols	Values	Units
Body mass	M	0.54	*kg*
Spring constant	K	193	*N/m*
Rest leg length	R	0.120	*m*
Pre-deflection	δ	0.024	*m*
Leg damping constant	B	0.35	*N s/m*
Leg inertia	J_{leg}	2.0×10^{-4}	$kg\ m^2$
Gravity acceleration	G	9.805	m/s^2
Stall torque	τ_{stall}	0.081	*N m*
No-load speed	ω_{nl}	3380	*rad/s*
Torque at current limit	τ_{cl}	0.020	*N m*
Total gear ratio	R	173	-
Torque transmission %	T	0.90^4	10^{-2} %

A motor and gear model is added in the simulation. In order to have a realistic result, it is imperative to limit the available torque at the hip joint based on the performance of the motor used on the actual robot [4]. The available hip torque of the SLIP is within its torque-speed performance curve. For details, refer to [4].

B. *Methods*

The simulation is done using the Matlab and Simulink software packages. The adopted integrator is a 4th-order integrator using Runge-Kutta method. Variable step time is used for the integration. The maximum step size and the tolerance are of both $1\times10^{-3}\ s$.

One simple criterion adopted to assess the stability of the running gait is to check whether the phase plot for the COM height is a limit cycle. At the same time, it is checked whether the toe clearance is high enough.

Another criterion is to check the size of the region of attraction for a desired steady state. Many initial conditions (IC) are tried in order to see how robust the whole robot of the SLIP kinematics and the controller is. The COM height and the forward speed at apex are used as the state of the robot at which its stability is checked. This 2D cross-section is a Poincaré section, and a collection of the initial points at the Poincaré section that converge to a certain cyclic hopping condition in 5 *s* is obtained as a region of attraction.

IV. SIMULATION RESULTS

The simulation results in stable and cyclic running. Several plots are shown so as to see the large possibility of the SLIP hopping robot.

A. *Motion in the Sagittal Plane*

Given an IC of a forward speed of 0.80 *m/s*, a vertical position of 0.180 *m* and a vertical speed of 0 *m/s*, then the model smoothly converges upon cyclic motion, as shown in Fig. 2. The state at the 14th apex (about 5 *s*) is forward speed 0.63 *m/s* and vertical position 0.171 *m*. This result indicates that the realization of a SLIP using only one actuator with the limited motor performance is possible if a proper IC is given.

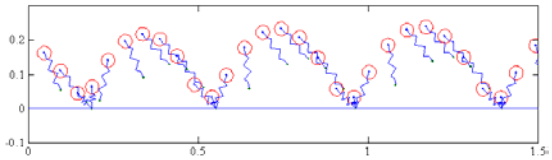

Figure 2. Hopping motion in the Sagittal Plane

B. *Phase Plot*

The phase plot for vertical position of the COM, i.e. vertical speed versus vertical position, is shown in Fig. 3. The IC is a forward speed of –0.60 *m/s*, a vertical position of 0.260 *m*, and a vertical speed of 0 *m/s*. The plot turns out to be a closed orbit, i.e. a limit cycle. One round designates one hopping cycle, and therefore, hopping height converges quickly and is stable. The approximate apex height in Fig. 3 shows that the toe clearance is about 0.050 *m* since the leg length in the flight phase is 0.120 *m*.

C. *Region of Attraction*

Fig. 4 shows how one IC converges to a stable state. The same IC as for Fig. 3 is used. The state at every apex is logged and plotted so that its transition can be seen.

We have tried 2115 points as IC's in order to illustrate a region of attraction as shown in Fig. 5. Two solid lines are drawn at 0.170 *m* and 0.8 *m/s*. A circle dot (●) means that the IC point converges into the desired area of apex height between 0.169 and 0.173 *m* and apex forward speed between 0.61 and 0.65 *m*. This height range is as desired. This forward speed range is lower than desired but results in approximately 0.8 *m/s* when averaging over the whole cycle. This is due to the forward speed in the stance phase, which is faster than 0.8 *m/s*. A plus mark (+) means that the leg length is shorter than 0.030 *m* at some point in 5 *s*, though the leg length does not become 0 *m* and the point converges to the desired area. This value comes from the physical limit of the actual robot structure. A cross mark (×) means that the IC does not converge to the desired area. It is seen in Fig. 5 that the IC's over a large domain converge to cyclic hopping, even with such a large initial error as negative forward speed –0.5 *m/s*. Thus, small disturbances from the actual environment will not destroy the cyclic hopping motion, indicating that a robust hopping robot can be realized using this design.

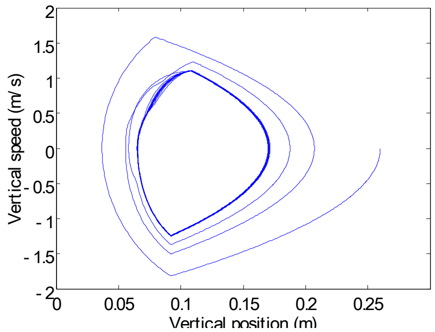

Figure 3. Phase plot for the vertical position of the COM for the IC of $\dot{x}=-0.60\,m/s$, $z=0.260\,m$, and $\dot{z}=0\,m/s$

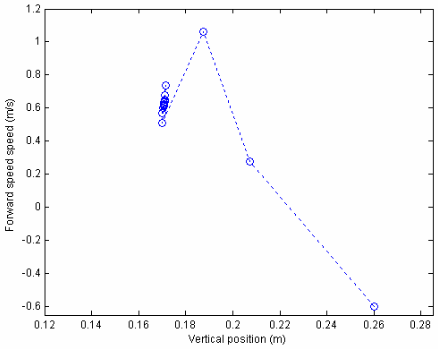

Figure 4. Transition plot of apex state starting with the IC of $\dot{x}=-0.60\,m/s$, $z=0.260\,m$, and $\dot{z}=0\,m/s$

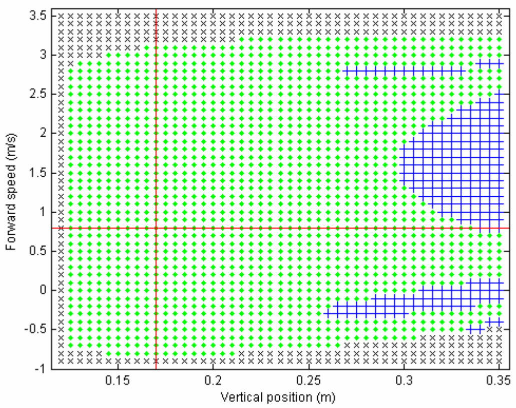

Figure 5. Region of attraction in simulation

D. *Summary*

A summary of the simulation results is as follows:

- Only one actuator and an unactuated compliant leg are sufficient to stabilize a SLIP robot (one of the SLIP model's properties)
- The determined mechanical parameters are good
- Specifications of the selected DC motor are sufficient
- A hybrid control scheme of two PD controllers is sufficient and robust.

In the simulation, it is confirmed that the SLIP-based design of the mechanical parameters and the controller is feasible.

V. EXPERIMENTAL SETUP

A. *Mechanical Structure*

Fig. 6 (a) shows the CAD design of our robot, the SLIP Hopper, with a planarizer. The planarizer consists of two beams and a rotation base and is needed for approximating sagittal plane motion. It also constrains the pitching motion of the body in order to conceptually place the COM of the body at the hip joint axis. Fig. 6 (b) shows the entire view of the actual setup of the robot with a planarizer. Fig. 7 shows a close view of the robot. The springy leg is realized as an unactuated prismatic joint with an extension spring. The lower leg shaft slides along in the upper leg bushing. There is only one actuated DOF at the hip. There is one motor with four-stage gears and a potentiometer in the black servo casing behind the white upper leg bushing. On top, an aluminum block is attached. Sensors are on the robot and the planarizer. Sensor configuration can also be checked in Fig. 8 for the next section.

B. Computer & Electronic Sytem structure

Two computers are used: the host computer is used for programming and compiling code and the target computer is for running the compiled code in order to control the robot. The sample rate for control is 1 *ms*. The robot is tethered to the target computer. The robot has a potentiometer for hip angle and an infrared sensor for ground detection. Two potentiometers are on the planarizer to measure the position in the constraint plane.

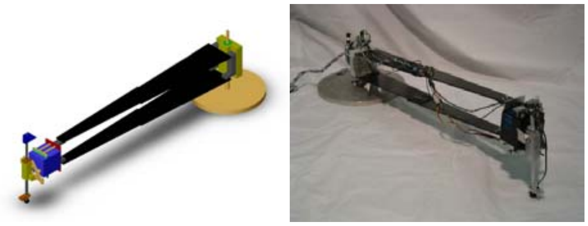

Figure 6. Entire view of (a) the CAD design and (b) the actual setup of the robot, the SLIP Hopper, with a planarizer

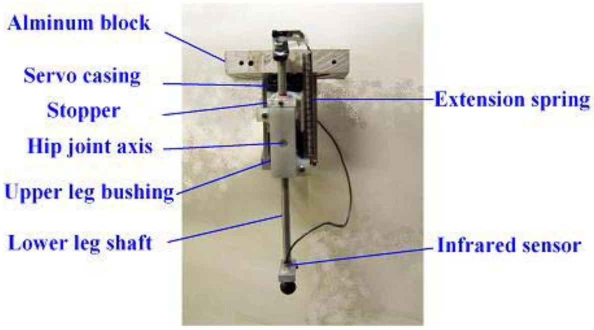

Figure 7. Close view of the SLIP Hopper

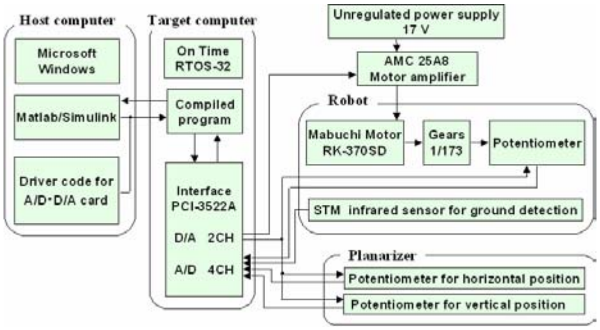

Figure 8. Computer & Electronic System structure

C. Methods

The same methods as for the simulation are adopted in order to see the stability of the robot's hopping gait. The experimental results for 10 IC's are obtained in the transition plot to check convergence. Unfortunately, we do not obtain an experimental region of attraction since the IC's are produced by releasing the robot in the air by hand, and it is impractical to produce 2115 IC's as in Fig. 6.

VI. EXPERIMENTAL RESULTS

Experiment on the SLIP Hopper successfully results in stable hopping that is similar to the simulation results.

A. Motion in the Sagittal Plane

Fig. 10 shows sequential snapshots from high frame-rate video. Each interval is approximately 30 *ms* while the frame rate is 250 *frames/s*. Note that the robot moves toward the left in the pictures as time progresses. The first snapshot starts at an apex achieved in the flight phase in a steady-state gait and the last one ends at the following apex. The sequence designates that the dynamical state of the robot at the first apex returns to the same state at the second apex so that the gait can repeat itself indefinitely.

Fig. 9 shows the locus of the COM from a set of experimental data with the IC of forward speed –0.58 *m/s*, vertical position 0.263 *m*, and vertical speed 0 *m/s*. Even with the initial negative forward speed, the robot starts running forwards after the first stance phase and its apex height converges to the desired height.

B. Phase Plot

Fig. 11 shows the phase plot for vertical position from the same set of experimental data as in the previous section. A stable limit cycle is obtained.

C. Region of Attraction

As explained in the method section, only the transition plot of apex for 10 IC's is obtained as in Fig. 12. All the 10 IC's go to the approximately same desired area as in the simulation, and it indicates a large region of attraction as the simulation result in Fig. 5.

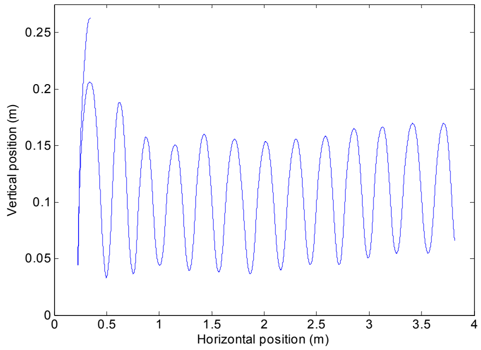

Figure 9. Vertical position versus horizontal position of the COM for the IC of $\dot{x} = -0.58\,m/s$, $z = 0.263\,m$, and $\dot{z} = 0\,m/s$

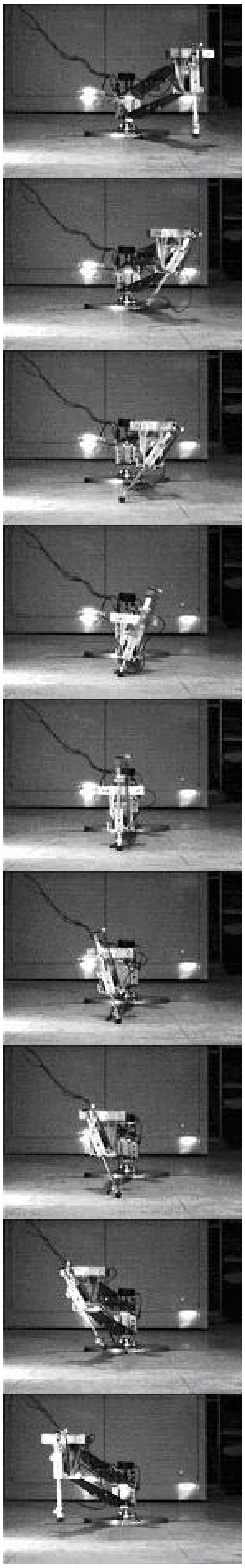

Figure 10. High frame-rate snapshots of hopping motion from an apex to the next apex

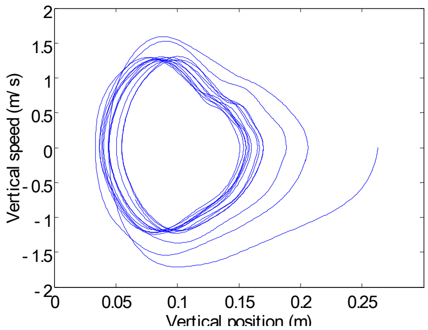

Figure 11. Phase plot for vertical position

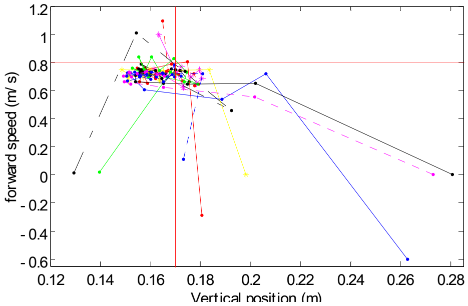

Figure 12. Transition plot of apex state for 10 IC's

VII. CONCLUSIONS & FUTURE WORK

A set of SLIP-model parameters and running controller is designed and validated, showing that there is no need of a more complex dynamics model. Using only one actuator and selecting appropriate touchdown and takeoff leg angles, an energy level is maintained and a robust gait is achieved in simulation and experiment.

We suggest future work to examine different mechanical parameters and different running gaits for the complete empirical validity of the SLIP model. Speed control for different desired speeds and height control for different desired heights are also needed for more practical applications.

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