IEEE Int. Conf. Robotics and Automation (ICRA), Leuven, Belgium, May 1998.

Disturbance Attenuation and Load Decoupling with $\mathcal{H}^\infty$ Positive Joint Torque Feedback

F. Aghili and M. Buehler
Dept. of Mechanical Engineering
McGill University
Montréal, QC H3A 2A7, Canada

J. M. Hollerbach
Dept. of Computer Science
University of Utah
Salt Lake City, UT 84112, USA

Abstract

Positive joint torque feedback can compensate the detrimental effects of load torques on position tracking performance. However, with (real world) non-ideal torque sources, simple unity gain positive torque feedback can actually deteriorate the performance, or even result in instability. An $\mathcal{H}^\infty$ joint torque feedback is proposed which takes the dynamics and uncertainty of the actuator into account and minimizes the system's sensitivity to load torques. In our experimental direct-drive system, the $\mathcal{H}^\infty$ positive torque feedback inner loop drastically improved the disturbance attenuation (25dB) and load decoupling properties of a simple PID position controller.

1 Introduction

The need for high performance motion control is pervasive in industrial applications, for example in automation, high speed tracking and pointing systems, machine tools, welding, laser cutting, or robotics. Once the performance of such systems reaches the limit of model based approaches, either because accurate models are difficult to obtain, are time-varying, or simply are not available, additional sensors might have to be added to achieve the required performance. In this paper, we examine direct-drive systems which are endowed with built-in load torque sensing [3]. We show how the torque information can be used to reduce the controller complexity considerably, compared to model based approaches, via *positive joint torque feedback*, and how disturbance attenuation and load decoupling can be achieved.

Positive joint torque feedback can, in theory, eliminate the effect of load torques on the motion servo completely if it is measured accurately, and then precompensated [2, 8]. In direct-drive systems, where a torque sensor is mounted between the motor's rotor and load, the (trivial) rotor dynamics remains the only plant dynamics to be controlled [2]. Kosuge [8] proposed a simple but effective control law for a two DOF SCARA direct-drive robot with joint torque sensors. Past research assumed the actuators to be ideal sources of torque and thus they could compensate load torques exactly via unity gain positive feedback. However, in practice, the actuator dynamics is not ideal, but has finite bandwidth. As a result, our experiments have shown that positive torque feedback may actually deteriorate the performance or even destabilize the system. This fact motivated the formulation of an optimal filter for positive joint torque feedback control which takes the actuator's dynamics into account and minimizes the sensitivity to load torques.

In this work we formulate joint torque feedback in the presence of actuator dynamics for the single variable case, yet the analytical solution can be extended for the MIMO case [1]. The paper is organized as follows. The optimal torque feedback which minimizes the torque disturbance sensitivity is derived in Section 2. Section 3 addresses the analysis and design of torque feedback when the load torques are generated via an external dynamical system, e.g. in robot manipulators. Section 4 reformulates and solves the problem of an optimal torque filter to comply with model uncertainty and maximum filter gain specifications. Finally, Section 5 demonstrates experimental results which validate the superior performance of our optimal torque feedback over conventional torque feedback.

2 Disturbance Attenuation

2.1 Sensitivity without torque feedback

The general block diagram for a motion servo loop is shown in Fig. 1 where $C(s)$ and $H(s)$ are the transfer functions of the controller and the actuator, and $M(s)$ represents the transfer function of the plant to be controlled. The system output, the actuator angle θ, should track the reference input r even in the presence of external torque disturbances τ_s. The input-output behaviour of the system is described by

$$\theta(s) = \begin{bmatrix} T(s) & -S(s)M(s) \end{bmatrix} \begin{bmatrix} r(s) \\ \tau_s(s) \end{bmatrix} \qquad (1)$$

where $S(s)$ and $T(s)$ are so-called sensitivity and complementary sensitivity transfer functions, and

$$S(s) = \frac{1}{1 + C(s)H(s)M(s)}, \quad T(s) = 1 - S(s). \quad (2)$$

Since we will be interested in the sensitivity of the closed-loop systems to external torque disturbances, we define

$$S_d(s) = -S(s)M(s). \qquad (3)$$

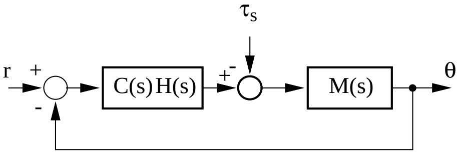

Figure 1: A typical motion control system

2.2 Without Position Feedback

A positive joint torque feedback system is shown in Fig. 2, where $H(s)$ represents the motor dynamics, $Q(s)$ the torque feedback filter, and $M(s)$ is the (unsensed) mechanical dynamics of the motor between the source of the motor torque and the torque sensor. For direct-drive motors with torque sensor mounted on the output shaft, $M(s)$ is simply the rotor dynamics, which becomes the plant to be controlled. The net torque acting on the rotor is $\tau_n = \tau_d - \tau_s$, where τ_d is the driving motor torque which is not directly measurable. τ_s is the external joint torque (the disturbance), which is measured via the torque sensor installed between the rotor and the load and which is fed back to the actuator for compensation through a filter $Q(s)$. As a first step, we shall ignore the load dynamics and the position feedback loop (dashed lines Fig. 2), and consider the problem of finding the optimum filter $Q(s)$ which minimizes the effect of the disturbance. The disturbance transfer function is

$$\chi(s) \triangleq \frac{\tau_n(s)}{\tau_s(s)} = -1 + H(s)Q(s). \qquad (4)$$

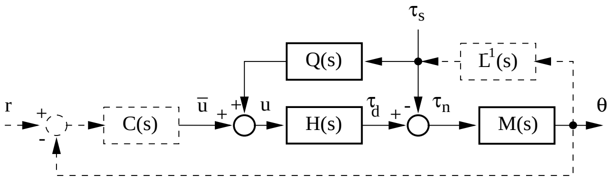

Figure 2: Positive joint torque feedback system

Our criterion is to minimize the worst-case disturbance attenuation, which is equivalent to selecting an optimal filter in the sense of $\mathcal{H}^\infty$. Typically, mechanical systems like actuators are strictly proper because they don't have any response at infinite frequency, $H(j\infty) = 0$. Hence, $\|1 - H(s)Q(s)\|_\infty \geq 1$, and by selecting trivially $Q(s) = 0$ the minimum is achieved. This implies that the disturbance sensitivity may get worse by any kind of torque feedback if the frequency is not restricted. Let $W_1(s) \in \mathcal{H}^\infty$ be a weighting function whose magnitude weighs the attenuation of the disturbance gain over frequency. Now we reformulate the problem to find a stable and realizable filter $Q(s) \in \mathcal{RH}^\infty$ ($\mathcal{RH}^\infty$ denotes the class of $\mathcal{H}^\infty$ functions which are rational) such that the maximum weighted sensitivity of the system $\|W_1(s)\chi(s)\|_\infty$ is minimized. The motivation for $\mathcal{RH}^\infty$ is that the resulting controller will be rational and easy to realize as a physical system. Suppose the maximum disturbance is

$$\mu \triangleq \sup_{s>0} |W_1(s)(1 - H(s)Q(s))|,$$

then the optimal filter $Q(s)$ minimizes

$$\mu_{opt} = \inf_{Q(s)\in\mathcal{RH}^\infty} \sup_{s>0} |W_1(s)\chi(s)|. \qquad (5)$$

This is a model-matching problem, and algorithms to compute the optimal $Q(s)$ are available [6]. It may be worth noting that the maximum gain of $W_1(s)$ plays no role in finding the optimal Q because it can be always factorized in (5). Hence for convenience we normalize $W_1(s)$ hereafter such that $\|W_1(s)\|_\infty = 1$.

In motion control applications, θ is the output of main interest. Therefore, we define the disturbance sensitivity from the disturbance input τ_s to the output θ in the frequency domain,

$$S_t(s) \triangleq \frac{\theta(s)}{\tau_s(s)} = -M(s)(1 - H(s)Q(S)) = M(s)\chi(s). \qquad (6)$$

This is equivalent to the previous model-matching problem (5), if $M(s)$ is chosen as the weight function

$W_1(s)$. Typically,

$$M(s) = \frac{\theta(s)}{\tau_n(s)} = \frac{1}{Js^2 + bs}, \qquad (7)$$

where J and b are the rotor inertia and the viscous friction in the joint bearing. Therefore, $M(s) \notin \mathcal{H}^\infty$ because it is unbounded at zero frequency and the problem (6) does not have an optimal solution in the sense of $\mathcal{H}^\infty$. That is, joint torque feedback alone cannot achieve torque disturbance attenuation in the $\mathcal{H}^\infty$ sense. Next we investigate the disturbance attenuation with combined torque and position feedback.

2.3 With Position Feedback

Now, the overall system sensitivity to torque disturbance, $S_t(s)$, is derived when both position and torque feedbacks are applied. We assume there is no correlation between the net torque τ_n and the sensor torque τ_s. Practical examples are applications in machine tools for metal cutting, slow robots for contour grinding, operating forces in precision index machines, or wind forces in tracking radar antennas. From Fig. 2 we have

$$\theta(s) = M(s)\chi(s)\tau_s(s) - C(s)\theta(s),$$

and,

$$S_t(s) \triangleq \frac{\theta(s)}{\tau_s(s)} = S_d(s)\chi(s). \qquad (8)$$

Hence, $\|S_t(s)\|_\infty = \|S_d(s)\|_\infty \|W_1(s)\chi(s)\|_\infty$, where $W_1(s) = S_d(s)/\|S_d(s)\|_\infty$. For a stabilizing controller $C(s)$, $S_d(s) \in \mathcal{H}^\infty$. Moreover, the disturbance sensitivity of the position feedback, $S_d(s)$, has large amplitude at low frequency and it decreases with frequency. Conversely, the magnitude of $\chi(s)$ is small at low frequency and it increases with frequency. Therefore, it can be concluded from (8) that the combined position and torque feedback makes disturbance attenuation over a wide frequency range feasible.

3 Decoupling from Load Dynamics

The above optimal torque feedback is well suited for systems in which no correlation exists between the load disturbance and system output. However, for tasks like free space motions of robot manipulators (laser cutting, pick and place operations) there is a dynamical relationships between the two signals.

Consider the rotor and load as a mechanical system. The input to the system is the net torque τ_n and the output is the torque sensor signal τ_s. Define the "rotor-load transfer function" $\Lambda(jw)$ such that its magnitude envelopes the ratio of the two signals,

$$|\Lambda(j\omega)| \geq \left| \frac{\tau_s(j\omega)}{\tau_n(j\omega)} \right| \qquad \forall \omega \in \mathbb{R}. \qquad (9)$$

In practice, it is reasonable to assume a finite gain for the transfer function, i.e. $\Lambda(s) \in \mathcal{H}^\infty$, due to internal damping [4], such that $\|\Lambda(s)\|_\infty = \lambda$.

When positive joint torque feedback is applied to a robot with ideal actuators, its link dynamics is completely decoupled and the remaining dynamics is the rotor dynamics [2]. However, a complete decoupling cannot be achieved in the presence of actuator dynamics. Our strategy then is to minimize the reflection of load dynamics on the nominal rotor dynamics.

Theorem 1 *The uncompensated load dynamics acts as a perturbation on the nominal rotor dynamics when positive joint torque feedback is applied. The maximum magnitude of the perturbation is minimized when the torque feedback filter $Q(s)$ is the model-matching solution of the weighted disturbance sensitivity in (6) where the weight function is the normalized rotor-load transfer function,* $\frac{1}{\lambda}\Lambda(s)$.

PROOF. Referring to Figure 2 we have,

$$\tau_n(s) = H(s)u(s) - \chi(s)\tau_s(s). \qquad (10)$$

Now by manipulating equations (4), (9), (7) and (10) we can derive the desired transfer function,

$$\frac{\theta(s)}{u(s)} = [1 + \Delta_G(s)]^{-1} G(s), \qquad (11)$$

where $\Delta_G(s) = \Lambda(s)\chi(s)$ and $G(s) = H(s)M(s)$. Provided $\|\Delta_G(s)\|_\infty \ll 1$ we can say,

$$[1 + \Delta_G(s)]^{-1} \approx 1 - \Delta_G(s). \qquad (12)$$

The upper bound for the uncertainty is

$$\|\Delta_G(s)\|_\infty = \|\Lambda(s)\chi(s)\|_\infty = \lambda \left\| \frac{\Lambda(s)}{\lambda}\chi(s) \right\|_\infty. \qquad (13)$$

Equations (11) and (12) show a perturbed system in which the perturbation enters as a multiplicative uncertainty for the nominal plant $G(s)$. Therefore, the problem of finding joint torque feedback $Q(s)$ minimizes $\|\Delta_G(s)\|_\infty$ is equivalent to the original model-matching problem (6). □

Theorem 1 guides us how to design the joint torque feedback when we are dealing with a dynamical load. The design requires a magnitude-frequency knowledge of the rotor-load transfer function $\Lambda(j\omega)$. In practice, the magnitude of the rotor-load transfer function has to be measured or estimated. Next, we estimate an upper bound on the perturbation when the joint torque feedback is synthesized based on the conservative estimate of $\Lambda(j\omega)$.

4 Two Block Formulation

The optimal torque filters derived above have a major practical drawback: their amplitude increases unboundedly at high frequency. However, the torque feedback is redundant at high frequencies because the inertia of the mechanical system naturally attenuates torque disturbances. In addition, the high gain produces a large control effort which can cause overheating in the power amplifier or the actuator. Therefore, it is desirable to roll off the torque feedback at high frequency where the magnitude of $S_d(j\omega)$ is sufficiently low. To accomplish this, we reformulate the problem in (5) to the following two block form,

$$\mu_{opt} = \inf_{Q\in\mathcal{RH}^\infty} \left\| \begin{bmatrix} W_1(1-HQ) \\ W_2Q. \end{bmatrix} \right\|_\infty \qquad (14)$$

where the magnitude of Q is penalized by the second weight function $W_2(s)$. The final improvement of this approach pertains to accommodating uncertainty in the actuator model, $H(s)$. Suppose the real actuator torque dynamics H_Δ is described in the form of a multiplicative uncertainty, $H_\Delta = H(1+\Delta)$, where the nominal transfer function $H(s)$ is accompanied by the perturbation $\Delta(j\omega)$. $\Delta(j\omega)$ can be any transfer function provided its magnitude is bounded by $\Delta_H(j\omega)$, i.e. $|\Delta(j\omega)| \le |\Delta_H(j\omega)|$. Then, the optimization problem in the presence of actuator modelling uncertainty is formulated as

$$\begin{aligned} \mu_{opt} &= \inf_{Q(s)\in\mathcal{RH}^\infty} \sup_{\Delta(s)} \sup_{s>0} |W_1(1-HQ) + W_1 H \Delta Q| \\ &\le \sqrt{2} \inf_{Q(s)\in\mathcal{RH}^\infty} \sup_{s>0} \left| \begin{bmatrix} W_1(1-HQ) \\ W_1 H \Delta_H Q \end{bmatrix} \right|. \end{aligned}$$

This is equivalent to problem (14) by setting $W_2(j\omega) = W_1(j\omega)H(j\omega)\Delta(j\omega)$.

5 Experiment

For the experiments, the McGill/MIT direct-drive motor [7], instrumented with a novel, hollow cruciform type torque sensor [3] is employed. A well tuned PID compensator is used for position control, and the actuator torque is provided via a ripple-free commutation law [5]. In the sequel, we shall compare the performance of the proposed $\mathcal{H}^\infty$ optimal controllers with the conventional unity gain feedback.

5.1 Disturbance Attenuation

The system is commanded to a follow a ramp reference signal while it is exposed to random torque disturbances. Since the tracking error due to the reference input signal is zero, the torque-position relationship is totally determined by the disturbance transfer function. The spectral analysis was performed on the time series of the output-input data, to estimate an empirical representation of the disturbance transfer function $S_t(s)$ and the actuator dynamics $H(s)$, shown as dashed lines in Fig. 3.

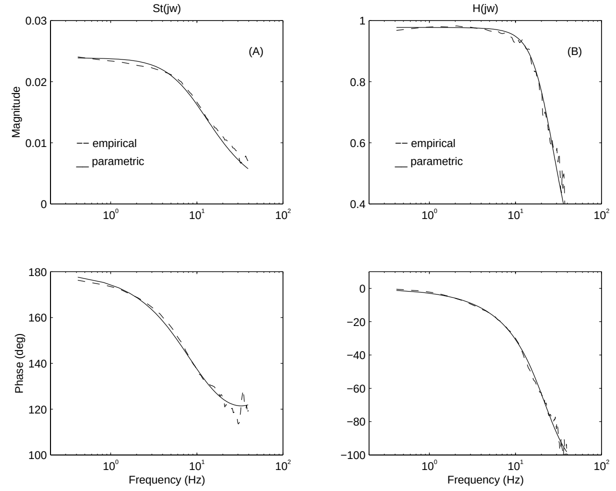

Figure 3: The empirical and parametric representations of $S_t(j\omega)$ and $H(j\omega)$.

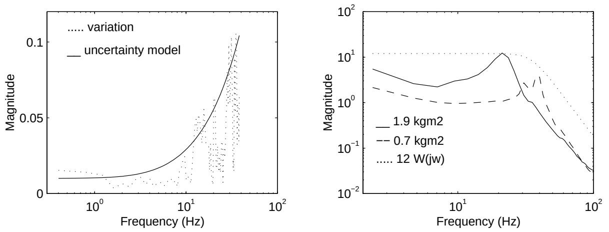

Figure 4: Uncertainty model $|W_2(jw)|$ (left), and graphs of $|\Lambda(jw)|$ (right).

The modeling error used to synthesize the $\mathcal{H}^\infty$ joint torque is

$$\Delta_H(j\omega) = \left| \frac{H_\Delta(j\omega)}{H(j\omega)} - 1 \right|$$

which is modeled as a first order function, shown as the solid curve in Fig. 4 (left). This is a conservative bound for the uncertainty which also extrapolates the uncertainty at high frequency where there is no experimental information.

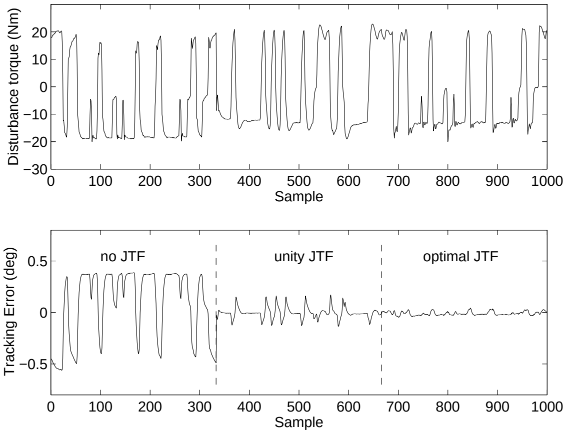

Figure 5: Position tracking error.

Fig. 5 illustrates the position tracking error trajectories to a ramp reference input in the presence of the torque disturbances when different torque feedbacks are applied. The control system exhibits relatively high disturbance sensitivity when there is no torque feedback. The conventional unity gain torque feedback is able to reject the slowly varying part of the torque disturbances, but not the higher frequency components. The tracking error is substantially reduced by $\mathcal{H}^\infty$ torque feedback.

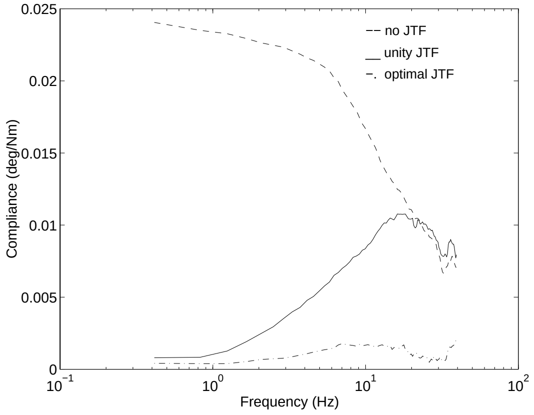

Figure 6: Experimental disturbance attenuation.

The disturbance attenuation performance is described more completely in the frequency domain (Fig. 6). As expected, the position servo system without torque feedback, $Q(s) = 0$, exhibits high sensitivity to disturbances within the loop bandwidth. It is evident that the system with unity gain torque feedback, $Q(s) = 1$, performs better than without any torque feedback, especially for low frequencies. However, at higher frequencies there is little improvement. The optimal feedback lowers the system sensitivity to disturbance remarkably over the whole frequency range. This validates the superior performance of the proposed joint torque feedback.

5.2 Dynamical Load

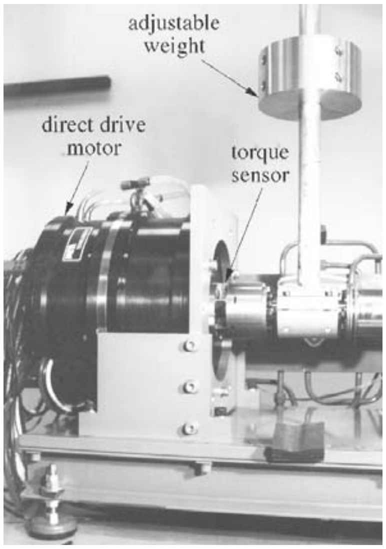

Figure 7: The single joint robot.

In this section, we demonstrate experimentally the performance and stability of joint torque feedback under dynamical loads, as encountered in many robotic applications. To this end, a link with a $7.2kg$ mass is mounted on the motor's torque sensor (Fig. 7). The weight can be mounted at different distances from the rotational axis to investigate the effect of an uncertain payload. As discussed in Sec. 3, the worst-case net torque to sensor torque transfer function, $\Lambda(j\omega)$ is required to design the optimal torque feedback which is derived as follows. The robot is commanded to move along a smooth trajectory, which is superimposed with a small white noise signal. The additive noise excites all mechanical modes to provide the input and output signals for the identification of $\Lambda(j\omega)$. Fig. 2 shows that the actuator signal, τ_d, can be reconstructed from the control input signal u and the identified actuator transfer function H. The magnitude of Λ obtained from experiments is shown in Fig. 4(right) for load inertias of 0.7 kgm^2 and 1.9 kgm^2.

Fig. 8 shows the tracking errors of the control system resulting from a sinusoidal reference signal, where the same PID position controller as in the previous experiments. First, no torque feedback is applied and large tracking errors result. In the next run of experiments we applied unity gain and the $\mathcal{H}^\infty$ torque feedbacks in addition to the PID position control law. Fig. 8 shows the tracking error due to the (unmodelled) nonlinearity gravitational torques of the $0.7kgm^2$ inertia load. The tracking error is reduced significantly when either unity gain or $\mathcal{H}^\infty$ joint torque feedbacks are applied. However, the unity gain joint torque feedback exhibits unstable behaviour when the load inertia is changed to $1.9kgm^2$, Fig. 8. In this case, the $\mathcal{H}^\infty$ joint torque feedback still has satisfactory performance which demonstrates the robustness of the proposed torque feedback.

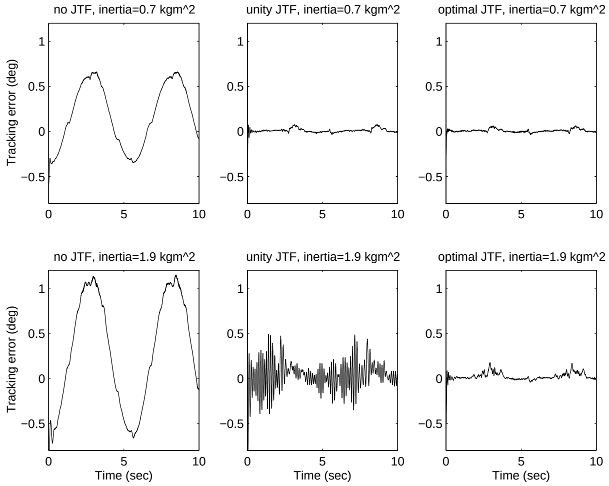

Figure 8: Position tracking error trajectories with no JTF (left), unity gain JTF (middle), and optimal JTF (right): experimental data.

6 Conclusion

The actuator's dynamics and uncertainty issues in the design of joint torque feedback have been addressed in this research. An optimal joint torque feedback, in the sense of $\mathcal{H}^\infty$, has been proposed to minimize the disturbance sensitivity. In the case of a dynamical load, the torque feedback optimally reduces the infinity norm of the perturbation to the nominal plant, the rotor dynamics.

The performance of joint torque feedback has investigated experimentally. The unity gain torque feedback cannot reject disturbances at high frequency because it does not compensate for the actuator dynamics. However, when the the actuator was cascaded with the optimal filter, a significant improvement, $25dB$, in sensitivity reduction was achieved. The joint torque feedback has been also implemented for a single joint robot. For low load inertias, both conventional unity gain and $\mathcal{H}^\infty$ torque feedbacks substantially reduced the tracking error brought by gravitational torque. However, the conventional unity gain torque feedback totally destablizes the system when the load inertia exceed a limit while the $\mathcal{H}^\infty$ feedback maintain stability and tracking accuracy.

Acknowledgements

This project was supported by the PRECARN TDS Project, through MPB Technologies of Montréal, Québec.

References

[1] F. Aghili. *Design and Control od Direct-Drive Systems with Applications to Robotics.* PhD thesis, McGill, October 1997.

[2] F. Aghili, M. Buehler, and J. M. Hollerbach. Dynamics and control of direct-drive robots with positive joint torque feedback. In *Proc. IEEE Int. Conf. Robotics and Automation*, pages 2865–2870, Apr 1997.

[3] F. Aghili, M. Buehler, and J. M. Hollerbach. A joint torque sensor for robots. In *ASME International Mechanical Engineering Congress & Exposition*, Dallas, TX, Nov 1997.

[4] F. Aghili, M. Buehler, and J. M. Hollerbach. Model-matching solution for optimal positive joint torque feedback. In *Proc. IEEE/RSJ Conf. Intelligent Systems and Robots*, Grenoble, France, Sept 1997.

[5] F. Aghili, M. Buehler, and J. M. Hollerbach. Optimal commutation laws for torque control of synchronous motors. In *American Control Conference*, Jun 1997.

[6] B. Francis. *Lecture Notes in Control and Information Science.* Springer, 1987.

[7] J. M. Hollerbach, I. Hunter, J. Lang, S. Umans, and R. Sepe. The McGill/MIT Direct Drive Motor Project. In *Proc. IEEE Int. Conf. Robotics and Automation*, pages 611–617, May 1993.

[8] K. Kosuge, H. Takeuchi, and K. Furuta. Motion control of a robot arm using joint torque sensors. *IEEE Trans. Robotics and Automation*, 6(2):258–263, 1990.