IEEE/RSJ Int. Conf. Intelligent Robots and Systems, IROS'97, Grenoble, France, Sept 1997.

Model-Matching Solution for Optimal Positive Joint Torque Feedback

F. Aghili and M. Buehler
Dept. Mechanical Engineering
McGill University
Montréal, QC H3A 2A7, Canada

J. M. Hollerbach
Dept. Computer Science
University of Utah
Salt Lake City, UT 84112, USA

Abstract

Applying positive joint torque feedback in direct drive motor systems reduces the effect of external load torques. This is most beneficial when accurate position tracking is required in the presence of large disturbances or for complex loads as found in robotic manipulators. We formulate the problem of torque feedback in the presence of actuator dynamics. The problem of finding the optimal torque feedback which minimizes the maximum disturbance sensitivity is equivalent to the model-matching problem. The effect of the load on the rotor dynamics appears as a perturbation minimized by the proposed torque feedback. To synthesize the torque filter, only the actuator dynamics and a very rough knowledge about the mechanical load is required.

1 Introduction

The performance of servo controllers depends on the accuracy of the plant model for which it was designed. It is well known that the performance degrades with increasing model uncertainty [13, 11]. Thus a common measure of controller performance has become its sensitivity to disturbances and model uncertainty. In this paper we are using this measure to design a robot controller. Robot manipulators can have very complicated dynamics since they are multibody systems driven by actuator-transmission systems which are difficult to model as well. This complexity makes it hard to implement model-based control [4]. Direct drive actuation eliminates the transmissions and permits more accurate dynamic models for control provided all the model parameters can be identified or are known [4, 5]. Furthermore, if direct drive systems are endowed with built-in torque sensing, this torque information can be used to reduce the controller complexity considerably via positive joint torque feedback.

Positive joint torque feedback can, in theory, eliminate the effect of the load on the motion servo completely if it is measured accurately, and then precompensated via an ideal source of torque [9, 2]. In this case the rotor of the direct drive motor is the plant to be controlled. The mechanical rotor dynamics is fairly simple and can be determined accurately [2]. In addition, in previous work we showed that we can compensate the nonlinearities of the motor using optimal commutation [3] based on online identification [1] of the motor parameters without any knowledge of the mechanical load.

However, the problem which stands in the way of compensating the load torques exactly is the actuator dynamics which has a finite bandwidth and may not respond fast enough to the load torque. In this case, positive torque feedback may actually deteriorate the performance or even destabilize the system. Since the actuator dynamics cannot be neglected, we propose an optimal filter for the torque feedback signal which minimizes the sensitivity of the system to the load torques. This is equivalent to the model-matching problem where a filter has to be cascaded to a given system such that the response of the whole system is as close to a desired response as possible.

Often the load torques are generated via an external dynamical system, as is the case in robot manipulators. In this case, we derive the entire system transfer function, under torque feedback. We show that the effect of the load dynamics enters as a perturbation to the nominal system, comprised of rotor and actuator dynamics. The optimal feedback minimizes the perturbation and makes the actual system close to the nominal system. Therefore an outer position control loop can be designed based on the simple nominal plant, independently of the load. The results of the paper and the performance of the proposed torque feedback are demonstrated based on the experimental servo amplifier response from our McGill/MIT motor testbed.

2 Problem Formulation

The block diagram of a direct drive motor system with positive joint torque feedback is shown in Fig. 1, where $M(s)$ represents the rotor dynamics, $H(s)$ the motor dynamics, and $Q(s)$ the torque feedback controller. In the absence of a transmission between motor and load, we can consider simply the rotor dynamics $M(s)$ of the direct drive motor as the plant to be controlled.

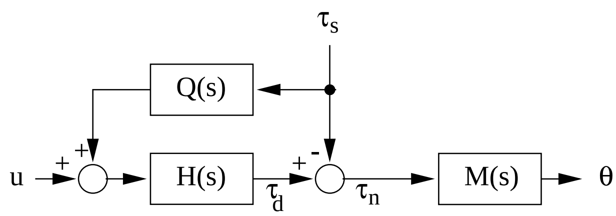

Figure 1: Positive joint torque feedback system

τ_d is the driving motor torque which is not directly measurable. However, the external joint torque, τ_s is measured via a torque sensor installed between the rotor and the load. It is important to note that motor torque cannot be achieved instantaneously, but is filtered by the actuator dynamics $H(s)$. In this case, the actuator dynamics $H(s)$ capture the dynamics of the driving motor torque, but does not include the mechanical rotor dynamics. It is here where our work differs from previous work in this field, which ignores the finite bandwidth of the actuator, i.e. assumes $H(s) = 1$, which then allows for simple unity gain positive joint torque feedback with $Q(s) = 1$.

Suppose the net torque acting on the rotor is τ_n,

$$\tau_n = \tau_d - \tau_s.$$

The disturbance (the external joint torque) τ_s is measured and fed back to the actuator for compensation through a filter $Q(s)$. Now the problem is to find the optimum filter which minimizes the effect of the disturbance, which is characterized by the disturbance transfer function,

$$X(s) = \frac{\tau_n(s)}{\tau_s(s)} = -1 + H(s)Q(s). \tag{1}$$

3 Optimal Feedback via Model Matching

Suppose the disturbance signal τ_s has finite energy, $\tau_s \in \mathcal{H}_2$. $W(s) \in \mathcal{H}_\infty$ is a weighting function, and its amplitude weighs the attenuation of the disturbance gain over frequency. Now the problem is to find a stable and realizable filter $Q(s) \in \mathcal{RH}_\infty$ such that the maximum weighted sensitivity of the system $\|W(s)X(s)\|$ is minimized,

$$\inf_{Q \in \mathcal{RH}_\infty} \|W - WHQ\|_\infty. \tag{2}$$

This is a model-matching problem, and algorithms to compute the optimal $Q(s)$ are available [6, 7]. It should be noted that the maximum gain of $W(s)$ plays no role in finding the optimal Q because it can be always factorized in (2). Hence for convenience we normalize $W(s)$ such that $\|W(s)\|_\infty = 1$. It should also be noted that in the case of non minimum-phase $W(s)$, it can be replaced by its outer factorization without any changes in the solution because the inverse of the inner factorization of $W(s)$ can be always taken out of the norm operator. Therefore, only right half-plane zeros of $H(s)$ must be taken into account.

Suppose γ is the maximum sensitivity to disturbance corresponding to an arbitrary filter $Q(s)$. Hankel operation is an elegant solution for the maximum attainable attenuation on the disturbance, γ_{opt},

$$\gamma_{opt} := \min \|W - WHQ\|_\infty.$$

The attenuation which determines the efficiency of the torque feedback method completely depends upon the location of right half-plane zeros of the actuator transfer function $H(s)$. For a minimum phase system the solution is trivial and the disturbances can be attenuated at will. But when $H(s)$ has a single zero in the right half-plane s_0, $\mathcal{R}e\ s_0 > 0$, according to the maximum modulus theorem [6] we can say $\gamma_{opt} \geq |W(s_0)|$ and $Q(s)$ has a unique solution ($|\cdot|$ indicates the magnitude of a complex variable). Since usually $W(s)$ is a low pass behaved function, it can be concluded, in general, that the right half-plane zero near the origin adversely restrict the functionality of joint torque feedback. In other words, the location of actuator right half-plane zeros is a rough indication of the frequency range of the disturbance rejection which can be achieved by any torque feedback.

4 Robustness

The introduced optimal torque feedback is well suited for systems in which no correlation exists between the load disturbance and system output. However, for systems like robot manipulators there is a dynamical relationships between the two signals. Consider the rotor and load as a mechanical system. The input to the system is the net torque τ_n and the output is the torque sensor signal τ_s. Suppose $\eta(s)$ shows how the net torque τ_n is transferred to the sensor torque τ_s through the external load and the rotor dynamics. We call this the "load transfer function"

$$\eta(s) = \frac{\tau_s(s)}{\tau_n(s)}. \tag{3}$$

This could be very complicated depending on the rotor dynamics and the external load dynamics, which can include compliant elements,

In practice, it is reasonable to assume a finite gain for the transfer function, i.e. $\eta(s) \in \mathcal{H}^\infty$, such that

$$\|\eta(s)\|_\infty = \alpha. \tag{4}$$

Suppose the bandwidth of the weighting function is chosen greater than that of the mechanical mode of the system. Since $\|W(s)\|_\infty = 1$,

$$|\eta(j\omega)| \leq \alpha\, |W(j\omega)| \quad \forall \omega \in \mathcal{R}, \tag{5}$$

which implies that $\alpha W(s)$ is an upper bound for $\eta(s)$ over all frequencies. This is shown graphically in Fig. 2. Although the load transfer function can change, for example due to changes in the load inertia, $W(s)$ can be selected to envelope all possible transfer functions.

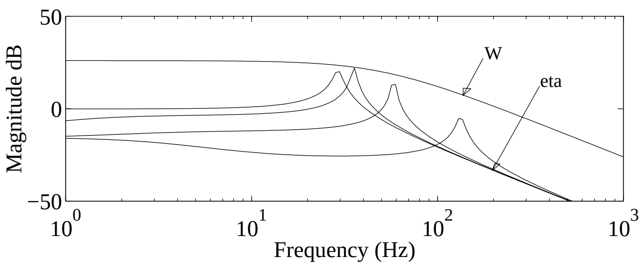

Figure 2: Graphs of $\alpha|W(j\omega)|$ and $|\eta(j\omega)|$, taken from the example in Sec.5.

The primary concern is the system stability with the closed torque feedback loop. According to the small gain theorem [12], the condition for stability is that the overall gain should be less than one,

$$\|\eta(s)X(s)\|_\infty \leq 1,$$

and with (5) we have

$$\|\eta(s)X(s)\|_\infty \leq \alpha \|W(s)X(s)\|_\infty. \tag{6}$$

Now we can state a more conservative but more comprehensive condition on the stability of torque loop,

$$\alpha \|W(s)X(s)\|_\infty = \alpha\gamma \leq 1. \tag{7}$$

Since $\gamma \leq \gamma_{opt}$, the optimal torque feedback in (2) provides the best gain margin. The condition (7) states that external loads with low damping require higher disturbance rejection for stability.

Next we investigate the stability of the closed torque feedback loop system with an outer position feedback loop. To this end we need to examine the open loop transfer function between $u(s)$ and $\theta(s)$ (see Fig. 1). Let the rotor transfer function be

$$M(s) = \frac{\theta(s)}{\tau_n(s)} = \frac{1}{J_r s^2 + c_r s}, \tag{8}$$

where τ_n is the net torque acting on the rotor, J_r and c_r are the rotor inertia and the viscous friction in the joint bearing. In a related paper, we derived the rotor dynamics for a serial link manipulator [3]. Furthermore, the rotor torque can be expressed in terms of both the input and exogenous signals as

$$\tau_n(s) = H(s)u(s) - X(s)\tau_s(s). \tag{9}$$

Now by manipulating equations (1), (3), (8) and (9) we can derive the desired transfer function,

$$\frac{\theta(s)}{u(s)} = [1 + \Delta(s)]^{-1}\, G(s), \tag{10}$$

where $\Delta(s) = X(s)\eta(s)$ and $G(s) = H(s)M(s)$. Provided $|\Delta(s)| \ll 1$ we can say,

$$[1 + \Delta(s)]^{-1} \approx 1 - \Delta(s). \tag{11}$$

Again from (6) we obtain

$$\|\Delta(s)\|_\infty \leq \alpha\, \gamma. \tag{12}$$

Equations (10) and (11) show a perturbed system in which the perturbation enters as a multiplicative uncertainty for the nominal plant $G(s)$. The nominal dynamics relies only on rotor and actuator dynamics and is independent of the load. Inequality (12) nicely reveals that the uncertainty is proportional to disturbance sensitivity. Consequently, the optimal torque feedback, given by (2), minimizes the perturbation and results in the most robust control system when the position control is only designed based on the nominal system. In practice γ is substantially reduced by optimal torque feedback such that $\alpha\gamma_{opt} \ll 1$. Therefore, the whole system can be controlled based on the nominal plant by an outer position feedback.

5 Selecting the Weighting Function

The objective of the weighting function $W(s)$ is to shape the power spectrum density of the disturbance. Therefore, only the magnitude $|W(s)|$ of the weighting function plays a role, not the phase. Typically, a physical system like an actuator is strictly proper because it doesn't have any response at infinite frequency, $H(j\infty) = 0$. Hence,

$$\|1 - H(s)Q(s)\|_\infty \geq |1 - H(\infty)Q(\infty)| = 1.$$

Consequently, by selecting trivially $Q(s) = 0$ the minimum is achieved. This implies that the disturbance sensitivity may get worse by any kind of torque feedback if the frequency is not restricted.

There are many cases where there is no correlation between the net torque τ_n and the sensor torque τ_s. Examples are applications in machine tools for metal cutting, operating forces in precision index machines, or wind forces in radar antennas. In this case, the best choice for $W(s)$ is the rotor transfer function, i.e $W(s) = M(s)$. Now the sensitivity of the whole system to disturbances is minimized when the optimal torque feedback is applied. On the other hand, when the two torques are correlated as in robot manipulators, a conservative estimate of $\eta(s)$ should be a $W(s)$ as shown in Fig. 2 and described in the previous section.

A simple example: In order to illustrate the behavior of $\eta(s)$, it is calculated for the simple system shown in Fig. 3.

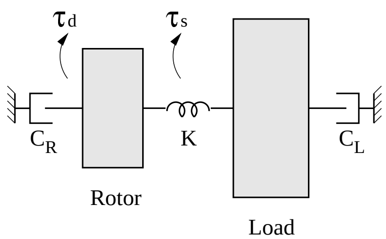

Figure 3: A simple mechanical system.

The sensor is shown as a torsion spring with stiffness k. Assuming a small deflection $\delta\theta$ in the torque sensor we can write the dynamic equations in the Laplace domain as

$$\begin{aligned} \tau_d - \tau_s = \tau_n &= (J_r s^2 + c_r s)\theta \quad (13) \\ \tau_s &= k \; \delta\theta \\ \tau_s &= (J_l s^2 + c_l s)(\theta - \delta\theta), \end{aligned}$$

where J_l and c_l are the load inertia and damping, respectively. Substituting θ from the last two equations above into the first, we can derive $\eta(s)$ as a third order system,

$$\eta(s) = \frac{k}{J_l s^2 + c_l s + k} \; \frac{J_l s + c_l}{J_r s + c_r}.$$

Fig. 2 shows the magnitudes of the above transfer function and a suitable weighting function when the ratio of the rotor to load inertias varies over three orders of magnitude, from 0.05 to 100.

6 Experiments and Simulation

In order to validate the effectiveness of the proposed torque feedback approach, we calculated the disturbance rejection factor γ based on a real system, the McGill/MIT direct drive motor [8]. To identify the motor's current amplifier dynamics, which can be close to the actuator dynamics $H(s)$ [3], a finite interval pseudo random signal was input to the system while the three phase currents were measured with a sampling rate of $2.5kHz$. The non-parametric modeling of the system was performed by spectral analysis. Then, several parametric models were examined, and it turned out that a fourth order system can match the behavior of the system adequately [10].

Fig. 4 demonstrates graphically the validation of the parametric system model. The loci of the poles and zeros of the system are plotted in Fig. 5 which clearly indicates that the system is non-minimum phase. An optimal filter $Q(s)$ based on the identified system and $30Hz$ disturbance bandwidth is synthesized.

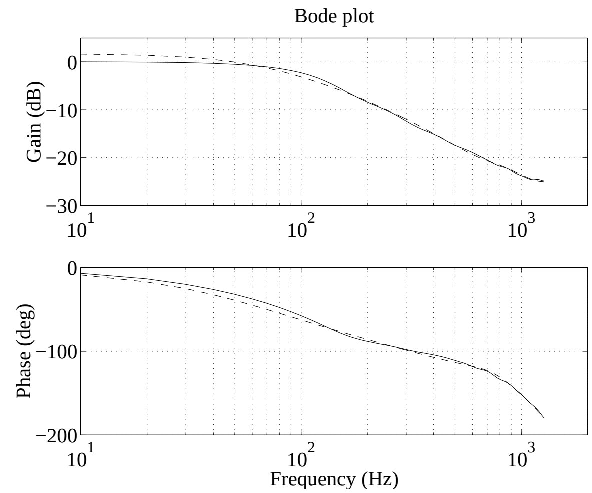

Figure 4: Frequency response of the motor's current amplifier. Solid and dashed lines show the non-parametric and parametric models, respectively.

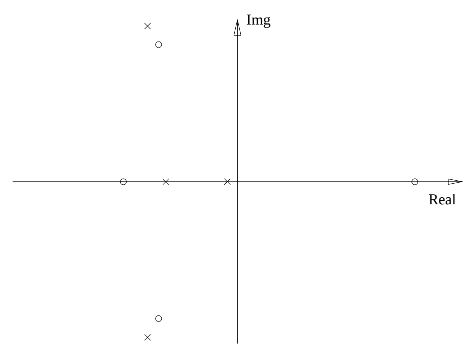

Figure 5: Zero (circle) and pole (cross) locations associated with the servo amplifier model in the s-plane. Results obtained from simulation.

Fig. 6 shows a comparison between the weighted disturbance transfer functions when the standard ($Q = 1$) and optimal torque feedbacks ($Q = Q_{opt}$) are applied. First, the dashed line shows the weighted disturbance sensitivity without any torque feedback ($Q = 0$), which is identical to the weighting function. It is evident that the system with the unit feedback performs better than without any torque feedback, especially for low frequencies. However, at higher frequencies there is little improvement. In contrast, the optimal feedback lowers the sensitivity to disturbance almost uniformly down to $-55dB$.

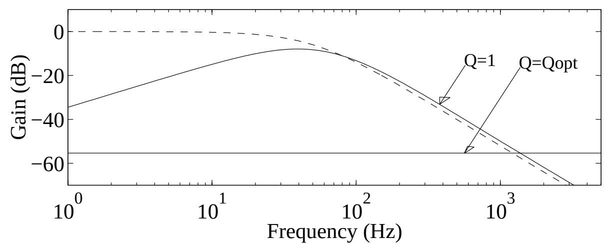

Figure 6: Magnitude of the weighted disturbance transfer functions, $|W(s)X(s)|$. The dashed line is the weighting function $W(s)$.

7 Conclusion

Direct drive motor control with positive joint torque feedback offers the promise of simplified control and high performance. With the external load torques measured and pre-compensated using a torque sensor, the remaining dynamics to be controlled are simply the motor's rotor dynamics. Unfortunately, in practice, the load torques cannot be compensated completely due to the motor dynamics which cannot be neglected. To address this problem, we proposed an optimal joint torque feedback in the H^∞ sense. The problem of finding the optimal filter turned out to be equivalent to the model-matching problem. When the load has dynamics, the problem of disturbance sensitivity reduction is equivalent to reducing the system perturbation when the load dynamics is ignored. In fact, the maximum perturbation is proportional to the maximum disturbance sensitivity. Consequently the optimal torque feedback offers the best robustness as well. We evaluated the performance of the torque feedback when the optimal and direct feedbacks were applied. The direct feedback cannot reject disturbances at high frequency because it does not compensate for the actuator dynamics. However, when the the actuator is cascaded with the optimal filter, a significant improvement in sensitivity reduction was achieved. Since the rotor dynamics is simple and decoupled from the environment, very precise position control can be accomplished. This hypothesis is currently being investigated on our motor test bed based on the McGill/MIT direct drive system. In addition, we will be able to measure directly the disturbance sensitivity under optimal torque feedback.

Acknowledgements

This project was supported by the PRECARN TDS Project, through MPB Technologies of Montréal, Québec.

References

[1] F. Aghili, M. Buehler, and J. M. Hollerbach. A new indirect adaptive control strategy for a synchronous direct drive motor. In *Proc. IEEE Int. Conf. Robotics and Automation*, pages 2865–2870, Minneapolis, MN, Apr 1996.

[2] F. Aghili, M. Buehler, and J.M. Hollerbach. Dynamics and control of direct-drive robots with positive joint torque feedback. In *Proc. IEEE Int. Conf. Robotics and Automation*, April 1997.

[3] F. Aghili, M. Buehler, and J.M. Hollerbach. Optimal commutation laws for torque control of synchronous motors. In *American Control Conference*, Jun 1997.

[4] C. H. An, C. G. Atkeson, and J. M. Hollerbach. *Model-Based Control of a Robot Manipulator.* MIT Press, Cambridge, MA, 1988.

[5] H. Asada and K. Youcef-Toumi. *Direct-Drive Robots.* MIT Press, London, 1987.

[6] C. Foias, B.Francis, J.W. Helto, H. Kwakernaak, and J.B. Pearson. *H infinity Control Theory.* Springer-Verlag, Berlin, 1990.

[7] B. Francis. *Lecture Notes in Control and Information Science.* Springer-Verlag, Berlin, 1987.

[8] J. M. Hollerbach, I. Hunter, J. Lang, S. Umans, and R. Sepe. The McGill/MIT Direct Drive Motor Project. In *Proc. IEEE Int. Conf. Robotics and Automation*, pages 611–617, May 1993.

[9] K. Kosuge, H. Takeuchi, and K. Furuta. Motion control of a robot arm using joint torque sensors. *IEEE Trans. Robotics and Automation*, 6(2):258–263, 1990.

[10] L. Ljung. *System Identification Toolbox for Use with MATLAB.* The Math Works Inc., Natick, MA, 1995.

[11] J.G. Owen. *Performance Optimization of Highly Uncertain System in H Infinity.* PhD thesis, McGill, Jaunuary 1993.

[12] J. Zames. On the input-output stability of time-varing nonlinear feedback system, part i: Conditions derived using concepts of loop gains, conicity, positivity. *IEEE Trans. Automatic Control*, 11(2):228–238, 1966.

[13] J. Zames. Feedback and optimal sensitivity: Model reference transformation, multiplicative seminorms, and approximate inverses. *IEEE Trans. Automatic Control*, 26(2):301–320, 1981.