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A New Indirect Adaptive Control Strategy for a Synchronous Direct Drive Motor

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Abstract

This paper presents a new model reference adaptive control method to control motor torque in direct drive reluctance motors in an accurate and ripple-free fashion. It is based on the motor's voltage equation and requires only measurements of motor angle, winding voltages and currents. In addition, we introduce a new optimal torque-to-current conversion which minimizes copper losses. Simulations of a reluctance motor with magnetically coupled phases demonstrate fast parameter convergence and accurate torque tracking.

Key words: *torque control, direct drive robots, parameter estimation, synchronous motor.*

1 Introduction

Direct drive robots whose joints are directly actuated by electro-mechanical motors without transmissions permit, in principle, precise and high-performance model based control [1]. However, due to non-sinusoidally distributed windings, which are the consequence of a high number of poles, and to the saliency of the air gap due to armature slots, in practice the generation of smooth torque is a major challenge in direct drive motor control. Most past research efforts on torque ripple cancellation as well as implementations in commercial drive units rely on feedforward compensation, implemented via a look-up table which translates the desired torque into the phase currents as a function of the motor angle [7, 4]. However, lookup table based methods have two significant disadvantages. First, since the table is generated off-line, this scheme can not easily cope with parameter changes caused by temperature changes. These can be significant during motor operation, especially in robotic applications, where the motor load can vary widely. As a result, the ripple compensation deteriorates. Second, each motor requires its own "finger-print" calibration which has to be done periodically if accuracy is to be maintained and certainly after each disassembly of the system. For this reason, online schemes have been investigated, which continuously estimate the motor model for purposes of accurate torque control.

Sepe [8] implemented an adaptive method to control the speed of a permanent-magnet synchronous motor. The parameters of the combined mechanical and electrical system were unknown. To make the problem tractable, the motor model was idealized, without cogging or torque ripples, and a simple mechanical load was considered. Chen [2] developed adaptive linearization for position control of a step motor. However it depends on a simple mechanical load and torque measurement. Shouse and Taylor [9] applied a self-tuning tracking controller for a permanent-magnet synchronous motor when both the electrical and mechanical parameters are unknown. Again the mechanical load is a pure inertia. Canudas De Wit [11] also designed a robust torque regulator for minimum energy consumption. However, the approach is based on induction motors and is limited to steady state operation.

A formidable challenge arises in the control of direct drive robots since at each joint a highly nonlinear multidimensional system, the motor, is coupled to another highly nonlinear multidimensional mechanical system, the robot. Therefore it is desirable to decouple the problem of identifying and controlling the motor from that of the robot. The adaptive method presented here achieves this by using the motor's voltage equation for motor parameter estimation and the torque equation to apply the correct current. The resulting accurate torque source can be utilized to identify the robot's mechanical dynamics and to control robot position, velocity or force independently of the actuator dynamics [3].

2 Dynamical model

The electrical dynamics of the system relating the terminal voltages V and winding currents i to the time varying magnetic flux linkage Ψ can be found according to Faraday's and Ohm's laws,

$$\frac{d\Psi(i,\theta)}{dt} = -Ri + V. \tag{1}$$

The electro-magnetic force can be found by the principle of virtual work,

$$\tau(i,\theta) = \frac{\partial W_c}{\partial \theta} = \frac{\partial}{\partial \theta}\int_0^i \Psi^T(\xi,\theta)d\xi \tag{2}$$

where W_c is the magnetic co-energy concentrated in the air gap [6]. These two equations govern the dynamics of most electro-mechanical actuators. The key point is that the parameter set of the second equation is a subset of the first one. This makes it possible to identify the required parameters by considering equation (1) only, which does not require any knowledge about the mechanical system except the motor position.

The actuator considered herein is an axial air gap wound field synchronous motor which has been determined as the optimal choice for direct drive motors [4]. In the following, a model based on previous work [7] for this type of motor in the presence of harmonics shall be developed.

For a linear electro-magnetic system the magnetic fluxes and currents are related by the inductance matrix:

$$\Psi(\theta, i) = \mathbf{L}(\theta)i \tag{3}$$

where the inductance and resistance matrices are

$$\mathbf{L}(\theta) = \begin{bmatrix} \mathbf{L_S} & \mathbf{L_{Sr}}(\theta) \\ \mathbf{L_{Sr}}^T(\theta) & \mathbf{L_r} \end{bmatrix}, \mathbf{R} = \begin{bmatrix} \mathbf{R_S} & 0 \\ 0 & \mathbf{R_r} \end{bmatrix}.$$

Note that the inductance matrix consists of self $(\mathbf{L_S}, \mathbf{L_r})$ and mutual inductances $(\mathbf{L_{Sr}}(\theta))$, and the resistance matrices are diagonal: $\mathbf{R_S} = diag[r_s, r_s, r_s]$ and $\mathbf{R_r} = diag[r_r, r_r, r_r]$. The self inductance is independent of the mechanical angle θ between rotor and stator. On the other hand, the mutual inductance $\mathbf{L_{Sr}}$ is a periodic function of position and hence can be approximated by a truncated Fourier series,

$$\mathbf{L_{Sr}}(\theta) = \sum_{k=0}^{N} m_k \begin{bmatrix} c_k & c_k^+ & c_k^- \\ c_k^- & c_k & c_k^+ \\ c_k^+ & c_k^- & c_k \end{bmatrix} + n_k \begin{bmatrix} s_k & s_k^+ & s_k^- \\ s_k^- & s_k & s_k^+ \\ s_k^+ & s_k^- & s_k \end{bmatrix}$$

where $c_k = \cos(p\nu_k\theta), c_k^+ = \cos(p\nu_k\theta + 2\pi/3), c_k^- = \cos(p\nu_k\theta - 2\pi/3)$ and $s_k = \sin(p\nu_k\theta), s_k^+ = \sin(p\nu_k\theta + 2\pi/3), s_k^- = \sin(p\nu_k\theta - 2\pi/3)$. The ν_k are the major harmonics and p is the number of poles. The stator's self inductance matrix is given by

$$\mathbf{L_S} = \begin{bmatrix} L_{ls} + L_{ms} & -1/2L_{ms} & -1/2L_{ms} \\ -1/2L_{ms} & L_{ls} + L_{ms} & -1/2L_{ms} \\ -1/2L_{ms} & -1/2L_{ms} & L_{ls} + L_{ms} \end{bmatrix}$$

where L_{ls} is the leakage inductance and L_{ms} is the magnetizing inductance. The rotor's self inductance, $\mathbf{L_r}$, is obtained by replacing the subscripts s with r. The electrical angle γ is defined as

$$\gamma = \tan^{-1}(\frac{n_0}{m_0}).$$

In the case of linear magnetics, the torque equation (2) becomes

$$\tau = \frac{1}{2}i^T \frac{\partial \mathbf{L}(\theta)}{\partial \theta} i. \tag{4}$$

Change of coordinates to a reference frame which rotates at an arbitrary angular speed is a standard feedback linearization technique applicable to an ideal motor, i.e., with sinusoidally distributed windings [6]. Nevertheless, such a change of coordinate can simplify the model even in the presence of harmonics as is the case with our motor model. The transformation

$$\mathbf{K}_x = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\alpha_x & \cos(\alpha_x - \frac{2\pi}{3}) & \cos(\alpha_x + \frac{2\pi}{3}) \\ \sin\alpha_x & \sin(\alpha_x - \frac{2\pi}{3}) & \sin(\alpha_x + \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

is a projection, i.e. $\mathbf{K}^T = \mathbf{K}^{-1}$; any vector f_{abc} and its transformation f_{dqo} has the property

$$\|f_{abc}\|_2 = \|f_{dqo}\|_2. \tag{5}$$

For mathematical convenience, all quantities are transformed to a reference frame attached to the rotor, via the Park transformation ($\alpha_r = 0$ and $\alpha_s = -\theta$) [6]. The magnetic flux in the new coordinate is

$$\begin{bmatrix} \Psi_{dqos} \\ \Psi_{dqor} \end{bmatrix} = \begin{bmatrix} \mathbf{L_S}' & \mathbf{L_{Sr}}' \\ \mathbf{L_{Sr}}'^T & \mathbf{L_r}' \end{bmatrix} \begin{bmatrix} i_{dqos} \\ i_{dqor} \end{bmatrix} \tag{6}$$

where

$$\begin{aligned} \mathbf{L_S}' &= \mathbf{K_S}\mathbf{L_S}\mathbf{K_S}^{-1} = diag[L_s', L_s', L_{ls}'] \\ \mathbf{L}'_{\mathbf{Sr}}(\theta) &= \mathbf{K_S}\mathbf{L_{Sr}}(\theta)\mathbf{K_r}^{-1} \end{aligned} \tag{7}$$

and

$$\mathbf{L}'_{\mathbf{sr}}(\theta) = \sum_{k=0}^{N} m'_k \begin{bmatrix} \cos(\theta_k) & -\sin(\theta_k) & 0 \\ \sin(\theta_k) & \cos(\theta_k) & 0 \\ 0 & 0 & 0 \end{bmatrix} + \sum_{k=0}^{N} n'_k \begin{bmatrix} \sin(\theta_k) & \cos(\theta_k) & 0 \\ -\cos(\theta_k) & \sin(\theta_k) & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (8)$$

Note that $L'_s = L_{ls} + 3/2L_{ms}$, $m'_k = 3/2m_k$, $n'_k = 3/2n_k$ and $\theta_k = p(\nu_k - 1)\theta$. The torque equation now is

$$\tau = i^T_{dqos}(\frac{\partial \mathbf{L}'_{\mathbf{sr}}(\theta)}{\partial \theta})i_{dqor}. \quad (9)$$

Since the last column and row of $\mathbf{L}'_{\mathbf{sr}}$ are zero, i_o does not contribute in the torque equation. Hence, for simplicity we rename $i = [i_d, i_q]^T$ and $V = [v_d, v_q]^T$, and the inductance matrix $\bar{\mathbf{L}}'_{\mathbf{sr}} \in \mathrm{IR}^{2\times 2}$ consists only of the non-zero elements of $\mathbf{L}'_{\mathbf{sr}}$.

3 Torque control

3.1 Optimal torque-current translation

Since there are infinitely many solutions of transforming the scalar torque τ to the three stator currents i_s and the three armature currents i_r we can find the solution which minimizes the copper losses. Without loss of generality we assume $R_s = R_r$, therefore the objective function, considering 5, is

$$f = \|i_{abcs}\|_2 + \|i_{abcr}\|_2 = i^2_{ds} + i^2_{qs} + i^2_{dr} + i^2_{qr}$$

subject to

$$\hbar = a i_{ds} i_{dr} - b i_{qs} i_{dr} + b i_{ds} i_{qr} + a i_{qs} i_{dr} - \tau_d = 0. \quad (10)$$

τ_d is desired trajectory, and the coefficients a and b can be derived from the antisymmetric matrix Λ as

$$\frac{\partial \bar{\mathbf{L}}'_{\mathbf{sr}}(\theta)}{\partial \theta} = \mathbf{\Lambda} \quad (11)$$

where

$$\Lambda_{11} = \Lambda_{22} = a(\theta) = p\sum_{k=0}^{N}(-m'_k\nu_k \sin\theta_k + n'_k\nu_k\cos\theta_k)$$

$$\Lambda_{12} = -\Lambda_{21} = b(\theta) = p\sum_{k=0}^{N}(-m'_k\nu_k \cos\theta_k - n'_k\nu_k\sin\theta_k).$$

Introducing the Lagrangian multiplier μ for the constraint, we have

$$\nabla f + \mu \nabla \hbar = 0. \quad (12)$$

Due to the symmetry between rotor and stator parameters, the candidate optimal point could be $i_{qr} = i_{ds}$ and $i_{dr} = i_{qs}$; therefore

$$\nabla \hbar = \begin{bmatrix} a i_{qs} + b i_{ds} \\ -b i_{qs} + a i_{ds} \\ -b i_{qs} + a i_{ds} \\ a i_{qs} + b i_{ds} \end{bmatrix}, \nabla f = \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{qs} \\ i_{ds} \end{bmatrix}.$$

Substituting into (12),

$$\mu = \pm \frac{1}{\sqrt{\det \Lambda}}. \quad (13)$$

The Lagrangian multiplier satisfies the necessary condition for a minimum, i.e., $\mu \neq 0$. This yields two solutions for the stator currents:

$$\begin{aligned} \mu^+ \quad &\Longrightarrow \quad \frac{i_{ds}}{i_{qs}} = -\frac{a}{\sqrt{a^2+b^2}+b} = \rho \\ \mu^- \quad &\Longrightarrow \quad \frac{i_{ds}}{i_{qs}} = -\frac{1}{\rho}. \end{aligned} \quad (14)$$

Substituting the above in the torque equation (10) yields,

$$i_{qs} = \sqrt{\frac{\tau}{-b(1-\rho^2) + 2a\rho}}. \quad (15)$$

The two solutions are associated with positive and negative torque. μ^- switches i_{ds} and i_{qs} and also changes the sign of the denominator in (15). Thus, only one solution is acceptable depending on the sign of torque. It is interesting to note that in the case of no harmonics (ideal motor) where $a = 0$, we obtain $i_{qr} = i_{ds} = 0$ and $i_{dr} = i_{qs} = i^*$, which coincide with the conventional torque-current translation [6, 7, 4, 2].

3.2 Online parameter estimation

It has been shown that the optimal current is $i_r = \bar{\mathbf{I}} i_s$, where

$$\bar{\mathbf{I}} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Substituting i_r into (6) and (1) yields the electrical dynamics of the stator. It should be noticed that due to the exact symmetry between rotor and stator, the rotor side has the same equation,

$$\frac{d}{dt}(L'_s i_{dqs} + L'_{sr}(\theta)\bar{\mathbf{I}} i_{dqs}) = -R_s i_{dqs} + V_{dqs}. \quad (16)$$

The above is a multi-output nonlinear dynamical system in the position and currents. However, it can be put into the form of linear regression suitable for parameter estimation [10] by defining proper variables and a filtering operation.

For this purpose, define $\psi_k, \phi_k \in \mathbb{R}^2$ which can be interpreted as the projection of the current in a frame rotating with the speed of the kth harmonic:

$$\psi_k \stackrel{\text{def}}{=} e^{J\theta_k}\bar{\mathbf{I}}i_s, \quad \phi_k \stackrel{\text{def}}{=} -\mathbf{J}\psi_k \tag{17}$$

where $\mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and

$$L_s' \frac{di_s}{dt} + \sum_{k=0}^{N} (m_k' \frac{d\psi_k}{dt} + n_k' \frac{d\phi_k}{dt}) = -r_s i_s + V_s. \tag{18}$$

It can be seen that each term in the above equation is linear in terms of the unknown parameters. However, it still can not be used directly in practice because it requires differentiation of i_s, ψ_k and ϕ_k. To solve this problem, both sides of the above equation are convolved by a strictly proper stable filter $h(t)$

$$\begin{aligned} &\int_0^t \{h(t-u)\frac{d}{dt}[L_s' i_s(u) + \qquad (19) \\ &\sum_{k=0}^{N} m_k' \psi_k(u) + n_k' \phi_k(u)]\} du \\ &= h(t) * (-r_s i_s(t) + V_s(t)) \end{aligned}$$

where $*$ represents a convolution integral. By partial integration, the left hand side is

$$h(t-u)[L_s' i_s(u) + \sum_{k=0}^{N} m_k' \psi(u) + n_k' \phi(u)] \mid_0^t -$$
$$\int_0^t \{\dot{h}(t-u)[L_s' i_s(u) + \sum_{k=0}^{N} m_k' \psi(u) + n_k' \phi(u)]\} du \tag{20}$$

Since the filter $h(t)$ is stable, then

$$h(t)[L_s' i_s(0) + \sum_{k=0}^{N} (m_k' \psi_k(0) + n_k' \phi_k(0))] \to 0$$

as time evolves. Therefore, by adequate rearrangement of equation (20), it can be rewritten in linear regression form. $P \in \mathbb{R}^q$ is the vector of unknown parameters:

$$P = [r_s, L_s', m_0', n_0', ..., m_N', n_N']^T.$$

where $q = 2N+2$ is the number of parameters, and N is the numbers of harmonics. It should be noted that r_s and L_s' play no role in feed-forward torque control.

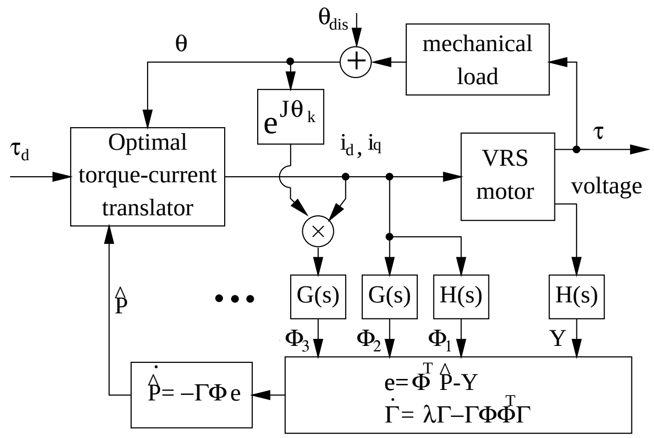

Figure 1: Structure of the adaptive torque control of the motor.

However, they have to be identified to extract the rest of required parameters. The regressors, $\Phi \in \mathbb{R}^{q\times 2}$, are

$$\Phi(t) \equiv \begin{bmatrix} h(t) * i_s(t) \\ g(t) * i_s(t) \\ g(t) * \psi_0(t) \\ -J[g(t) * \psi_0(t)] \\ \vdots \\ g(t) * \psi_N(t) \\ -J[g(t) * \psi_N(t)] \end{bmatrix}$$

where $g(t) = h(0)\delta(t) - \dot{h}(t)$, and $\delta(t)$ is the unit impulse function. The output $Y \in \mathbb{R}^2$ is

$$Y(t) = h(t) * V_s(t) = \Phi^T P.$$

The transfer functions of the filters $h(t)$ and $g(t)$ are $H(s)$ and $G(s)$, respectively. A simple choice of a permissible first order pair for the two transfer functions is

$$G(s) = \frac{\sigma s}{s+\sigma}, \quad H(s) = \frac{\sigma}{s+\sigma},$$

a high and a low pass filter. σ, which multiplies the differentiation may in practice be limited by the level of noise in the system. On the other hand, a very small value of σ reduces the bandwidth of the low pass filter $H(s)$ which may eliminate useful information from the input i_s and output V_s signals. This in turn decreases persistent excitation of the input signals. The best value for σ is a tradeoff which depends on practical information like noise characteristics. However, simulation results can be a useful tool to determine proper ranges.

The parameters of the system are affected mainly by a motor's thermal dynamics. Since thermal dynamics are slow compared to the electrical dynamics, a least squares method with forgetting factor is an appropriate technique to keep up with the parameter drift. Suppose $\hat{P}$ is the estimate of parameter vector P. Therefore the estimation and output errors are

$$\tilde{P} \equiv \hat{P} - P \qquad e = \Phi^T \hat{P} - Y.$$

The parameter update law is

$$\dot{\hat{P}} = -\mathbf{\Gamma} \Phi e$$

where $\mathbf{\Gamma} \in \mathbb{R}^{q \times q}$ is the gain of the estimator. This gain, in turn, can be found by

$$\dot{\mathbf{\Gamma}} = \lambda(t)\mathbf{\Gamma} - \mathbf{\Gamma}\Phi\Phi^T\mathbf{\Gamma}$$

where the scalar λ is the forgetting factor

$$\lambda(t) = \lambda_0(1 - \frac{||\mathbf{\Gamma}||}{k_0}).$$

For more details, the interested reader is referred to [10]. The proposed adaptive feed-forward torque controller is schematically shown in Figure 1. It can be seen that to identify each harmonic one filter acting on the input vector is required.

4 Simulation Results and Discussion

For simulation, the entire system was discretized. The true parameters of the motor and the major harmonics are chosen based on experimental results with the McGill/MIT motor [5]. We included the five main harmonics, namely $\nu = [1, 3, 6, 12, 24]^T$, resulting in 10 parameters to be identified. In addition the method requires the identification of two more parameters R_s and L'_s, termed “Resistance” and “Inductance” in Figure 2. In addition Figure 2 illustrates the convergence of the parameter estimates, $r_s, L_s, m_0, m_1, m_2, m_3$ to the true values. The mechanical load was a second order system

$$\frac{\theta(s)}{\tau(s)} = \frac{1}{Is^2 + Cs}.$$

Note that the convergence rate is very high even though the torque signal contains only one harmonic. The necessary rich signal is provided by the other input to the system, angular position, which is related to the torque command, depending on the nature of the mechanical load and the performance of the feed-forward controller. Since the controller initially performs poorly, in the absence of good parameter estimates, it introduces substantial torque ripple. This torque ripples in turn generate position trajectories which are rich in frequency content. After only a few seconds of operation all parameters converge to their true values. The tracking torque is shown in Figure 3. Notice that the motor failed to generate torque at the beginning. This is because the initial parameter estimates are assigned randomly and hence much of the contribution of the harmonic terms to torque may cancel. Figure 4 displays the power losses in the windings once the system tunes its parameters with the given reference torque trajectory. The amount of power saved by the optimal controller is shown at the bottom of the figure. Typically, the optimal translator loses 14% less power, which means that the motor is able to generate more torque. This is vital for direct drive applications. In general the amount of saved power depends upon the harmonics of the flux linkage with respect to angular position, or more precisely the diagonal ratio of matrix $\mathbf{\Lambda}$, a/b, in (11). Generally, the larger the ratio, the higher the power savings.

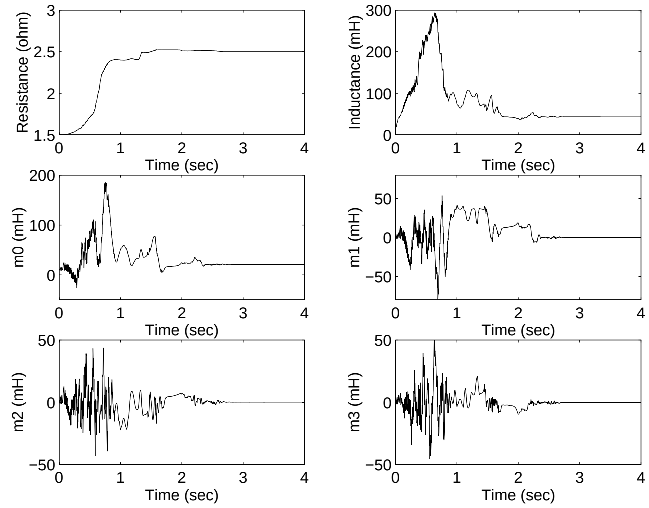

Figure 2: Estimated motor parameters.

Conclusion

Motivated by our experimental work on direct drive motor design and control, new methods for identifying the model parameters of direct drive reluctance motors and for converting a desired torque to phase currents are proposed. In conjunction with the estimator, an optimal torque translator which minimizes copper losses is analytically derived. The unique feature of this adaptive arrangement is that it does not rely on a motion control law or knowledge of mechanical loads which makes the whole system modular.

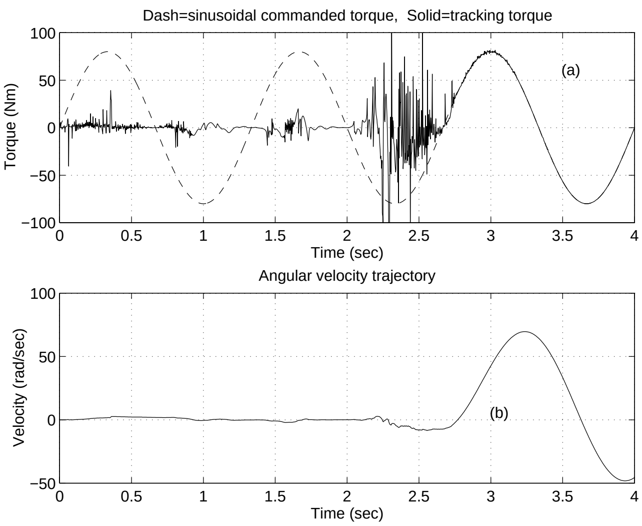

Figure 3: Simulation results of the adaptive torque control of the synchronous motor: (a) commanded and tracking torque; (b) angular velocity of the motor shaft.

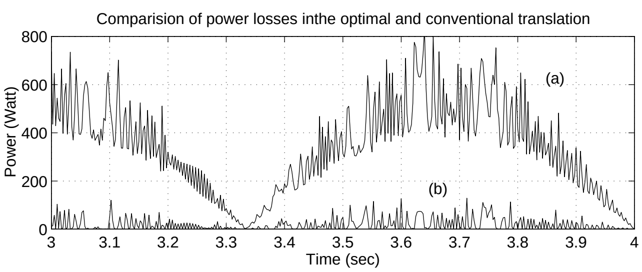

Figure 4: Copper losses in the optimal translator (a) and the savings over the conventional method (b).

The major advantage of our on-line torque controller over off-line look-up-table approaches is its ability to keep up with parameter drift. Another outcome of the identification is the instantaneous value of the coil resistance. Although superfluous for purposes of torque generation, this parameter could be employed to estimate the coil temperature based on the known resistance-temperature relationship. This idea is part of our currently ongoing implementation efforts.

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