

Control of Virtual Motion Systems

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Abstract

We introduce a new control problem: the control of motion simulating devices (Virtual Motion Systems, or VMS) for walking and running humans and robots in a fashion that feels most “realistic,” that is, like locomoting on ground. After developing simplified dynamical models for the VMS, the human/robot and the resulting coupled system, we cast the problem in terms of a performance index. This approach permits application of standard optimal control theory. We present two solutions and discuss upcoming problems in the task domain of virtual motion control.

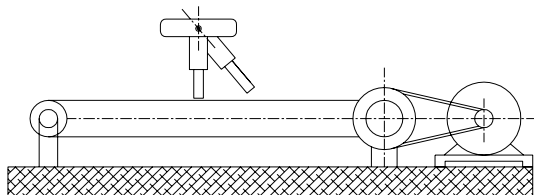


Figure 1: Legged robot on a treadmill

1 Introduction

Sensation in Artificial Reality environments could be considerably enriched if we could provide a realistic simulation of locomotion [6]. People could walk and run in any direction, without limitation, while guided by visual and auditory impressions from their head mounted displays. At the same time, however, they would not go anywhere, because they are moving on a “virtual motion device.” Since motion is restricted to a small physical space it becomes feasible to instrument it with sensors to

detect movement, and with various displays and actuators to provide external stimulation to simulate artificial environments.

If we replace the human in this system with a legged robot, we obtain an essential tool for systematic research in legged locomotion. Since it is difficult to work with free roaming robots due to sensing and space limitations, they often operate on treadmills. At constant speeds, the robot dynamics are identical to running on the ground. During transients, however, which are the subject of controller developments, there is dynamic interaction between the treadmill and the robot controller. This might change the robot controller performance significantly between treadmill and ground operation. Worse, the dynamic interaction could result in instability. To investigate these problems is the primary motivation for this work.

Before VMS systems become practical, three problems need to be solved. First, the planar equivalent of a treadmill needs to be developed, providing unrestricted planar motion. Solutions to this problem are presently being addressed within the artificial reality community. Iwata and Matsuda [1] have built a walk-through simulator, in which a human slides on omni-directional roller skates. In addition to feeling friction forces, strings attached to each skate permit the transmission of a certain amount of external stimulus. In this paper, we have limited ourselves to unidirectional motion on a treadmill. Second, we need to identify realistic models of the human and robot locomotion. As a first step, we have adopted the simplest models for locomotion and velocity control from the literature. Finally, we need to model the VMS itself, and develop control

laws which result in a stable coupled system, provide realistic interaction forces and assure that the human or robot remains within the bounds of the VMS system. We will present a solution to this last set of problems.

2 The Model

For the purpose of obtaining an analytically tractable system, we ignore for now the intermittent dynamical nature of the interaction between robot and VMS, and assume it possesses the simpler dynamics of a wheeled cart,

$$f_c = m\dot{v}_c \quad (1)$$

where v_c, m are the cart velocity and mass, respectively. That this is not an unreasonable assumption is demonstrated by the fact that the simplest legged locomotion models are based on spoked wheels [3, 4]. The ground force $f_c = \tau_h/r$ is generated by the robot’s hip torque τ_h and depends on the leg length r . In the sequel we will refer to the device on the VMS as the *cart*.

The cart’s velocity feedback controller will be modeled as

$$f_c = -\kappa_v \cdot (v_{cb} - v_{cbd}) \quad (2)$$

where v_{cb} and v_{cbd} are the actual and the desired velocity of the cart v_c with respect to the virtual ground v_b (i. e. treadmill belt), respectively. Again, this is a reasonable cart/robot control model. Existing velocity controllers for legged robots typically contain such a proportional error term (e.g. [5]).

In isolation, the VMS dynamics — in this case that of a treadmill — where a motor with angle θ and torque τ actuates a drive roller with radius r_d and inertia J , is given by

$$\tau = J\ddot{\theta} + r_d f_c = \frac{J}{r_d} \dot{v}_b + m r_d \dot{v}_c.$$

The resulting coupled cart–VMS system is a third order controllable system described by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u + \mathbf{e}w \quad (3)$$

where

$$\mathbf{x} = \begin{bmatrix} x_c \\ v_c \\ f_c \end{bmatrix}, \quad u = \tau, \quad w = \dot{v}_{cbd},$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1/m \\ 0 & 0 & -\kappa_v/m(1 + m r_d^2/J) \end{bmatrix},$$

$$\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \kappa_v r_d/J \end{bmatrix}, \quad \mathbf{e} = \begin{bmatrix} 0 \\ 0 \\ \kappa_v \end{bmatrix}.$$

3 Problem Formulation and Solution

The full problem can be formulated in the following fashion: Find a control input u to (3) with bounds

$$\|u\| \leq \hat{u} \quad (4)$$

which minimizes in some sense the interaction force and velocity profile difference to the cart on the ground

$$\min\{(f_c - f_g), (v_{cb} - v_g)\} \quad (5)$$

where

$$f_g = \kappa_v e^{-\alpha t}, \quad v_g = v_{cbd} \cdot (1 - e^{-\alpha t}), \quad (6)$$

with $\alpha = \kappa_v/m$. The “ground reference” (6) is obtained simply by solving the system described by (1) and (2) on the ground, with $v_{cb} = v_c$ and $v_{cbd} = v_{cd}$. For the desired cart velocities we assume a step input, starting with all initial conditions at zero. In addition, the position of the cart must remain on the VMS,

$$\|x_c\| \leq \hat{x}. \quad (7)$$

The very nature of the problem is that it is not possible to satisfy all of the above conditions simultaneously. For example, due to condition (7), the final cart velocity must be zero, but from (2) it is also the integral of force,

$$\dot{x}_c(t_f) = \int_0^{t_f} f_c dt = 0,$$

which *requires* an overshoot in the force profile for operation on the VMS — it is impossible attain an exponential force profile as desired by condition (5).

Since we need to minimize mutually contradictory performance measures an optimal control approach seems to be suitable and is pursued below.

3.1 LQR Approach

First, we apply the standard LQR optimal control solution. In order to accommodate a nonzero steady state $\mathbf{x}_d^* = [\hat{x} \ 0 \ 0]^T$ we compute the control offset u^* from the steady state equations

$$\begin{aligned} \mathbf{0} &= \mathbf{A}\mathbf{x}^* + \mathbf{b}u^* + \mathbf{e}w^* \\ \Delta y &= \mathbf{c}^T(\mathbf{x}_d^* - \mathbf{x}^*) \end{aligned} \quad (8)$$

with $\mathbf{c}^T = [1 \ 0 \ 0]$ and $\Delta y = 0$. Since we are commanding a step input, we have $w = 0$ as well. We can now solve the algebraic linear equation,

$$\begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{c}^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}^* \\ u^* \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{c}^T \mathbf{x}_d^* \end{bmatrix}$$

for \mathbf{x}^* and u^* .

Now, we can determine the LQR feedback controller $\Delta u = \mathbf{k}^T \Delta \mathbf{x}$ based on the performance index

$$J = \int_0^\infty (\mathbf{x}^T \mathbf{Q} \mathbf{x} + r u^2) dt$$

with the weights \mathbf{Q} and r .

Applying the simple LQR solution, however, comes at a price. Conditions (4) — (7) must be satisfied indirectly, via adjustments of the four scalars in \mathbf{Q} and r . Satisfactory results were readily obtained and are shown for

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10^{-4} \end{bmatrix} \quad r = 1.$$

The control input (Figure 2) remains within the desired bounds. Figure 5 shows the cart's position, which remains bounded and settles to the given maximum value with little overshoot. Furthermore, Figure 4 shows that the velocity of the cart with respect to the VMS remains close to its velocity when operating on the ground. Most importantly, the interaction force profile of the cart (Figure 3) remains close to its desired value when operating on the ground, while still exhibiting the necessary overshoot.

Using Matlab to determine the feedback gain matrix \mathbf{k} , we obtain the optimal control as $u = u^* - \mathbf{k}^T(\mathbf{x} - \mathbf{x}^*)$.

3.2 Reference Optimization

Now, we formulate the optimal control problem in a more general way that lets us express the conditions (4) – (7) explicitly in the performance index. The resulting numerical problem is much more complex and so far only converges for a simplified version of the VMS dynamics. Specifically, we suppose that the velocity of the VMS can be controlled directly, which becomes the new input to the system. Equations (2) and (1) are still valid and result in

$$\begin{aligned} m\dot{v}_c &= -\kappa_v(v_{cb} - v_{cbd}) \\ \dot{v}_c + \alpha v_c &= \alpha v_b + \alpha v_{cbd}. \end{aligned}$$

The state space form of these equation can be written as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u + \mathbf{e}w \quad (9)$$

where

$$\begin{aligned} \mathbf{x} &= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_c \\ v_c \end{bmatrix}, \quad u = v_b, \quad w = v_{cbd}, \\ \mathbf{A} &= \begin{bmatrix} 1 & 0 \\ 0 & -\alpha \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ \alpha \end{bmatrix}, \quad \mathbf{e} = \begin{bmatrix} 0 \\ \alpha \end{bmatrix}. \end{aligned}$$

We are interested in finding a control law $u(t)$ which minimizes the performance index

$$\begin{aligned} J &= \int_{t_0}^{t_f} \frac{1}{2} [q(f_c - f_g)^2 + w(v_{cb} - v_g)^2 \\ &\quad + s x_c^2 + t v_c^2] dt \end{aligned} \quad (10)$$

subject to (9), with f_c , the contact force, f_g, v_g , the contact force and cart velocity on the ground (6), v_{cb} , the cart velocity wrt the VMS, x_c, v_c the cart position and velocity q, w, s, t are scalar weights.

We now follow the derivation of an optimal control based on *Pontryagin's Maximum Principle* [2]. From the performance index (10) and the underlying cart–VMS dynamics (9), the Hamiltonian can be defined as

$$\begin{aligned} H &= \frac{1}{2} [q(f_c - f_g)^2 + w(v_{cb} - v_g)^2 \\ &\quad + s x_c^2 + t v_c^2] + \mathbf{p}^T \dot{\mathbf{x}} \end{aligned}$$

where \mathbf{p} contains the Lagrange multipliers (co-states). With

$$\dot{\mathbf{x}}^* = \frac{\partial H}{\partial \mathbf{p}}, \quad \dot{\mathbf{p}}^* = -\frac{\partial H}{\partial \mathbf{x}},$$

the conditions of optimality are

$$\begin{aligned} \dot{p}_1^* &= -sx_1^* \\ \dot{p}_2^* &= q\kappa_v[\kappa_v(w + u^* - x_2^*) - f_g] - tx_2^* \\ &\quad - w(x_2^* - u^* - v_g) - p_1^* + \alpha p_2^* \\ \frac{\partial H}{\partial u} &= q\kappa_v[\kappa_v(w + u^* - x_2^*) - f_g] \\ &\quad - w(x_2^* - u^* - v_g) + \alpha p_2^* = 0 \end{aligned}$$

and

$$\frac{\partial^2 H}{\partial u^2} \geq 0.$$

The optimal control then is given as

$$u^* = \frac{q\kappa_v^2(x_2^* - w) + q\kappa_v f_g + w(x_2^* - v_g) - \alpha p_2^*}{q\kappa_v^2 + w}.$$

Inserting u^* in the state equations, we can combine the state and co-state equations

$$\begin{aligned} \dot{x}_1^* &= x_2^* \\ \dot{x}_2^* &= \alpha(-x_2^* + w + u^*) \\ \dot{p}_1^* &= -sx_1^* \\ \dot{p}_2^* &= q\kappa_v[\kappa_v(w + u^* - x_2^*) - f_g] - tx_2^* \\ &\quad - w(x_2^* - u^* - v_g) - p_1^* + p_2^*\alpha. \end{aligned}$$

In order to solve this set of differential equations, with some initial and some final boundary values, it is necessary to use the shooting method. For this purpose, we have used the IMSL package subroutines. Again, as with the LQR method in the previous section, proper weighting factors need to be chosen to arrive at a satisfactory tradeoff between the conflicting goals. The best set of weights was determined to be

$$q = .01, \quad w = .01, \quad s = 12.5, \quad t = 1.4.$$

The results are displayed in Figures 6 — 8. Figure 6 and 7 show that both the interaction force profile and the cart velocity with respect to the VMS are very close to their counterparts on the ground. The position of the cart (Figure 8) exhibits an almost critically damped response toward the steady state of $1m$ – in practice the treadmill length. When

comparing these results to the ones from the LQR approach, it turns out that they are only marginally better, despite the superior translation of the problem specifications into the performance index and despite the much increased computational effort incurred with this optimization method.

4 Conclusion

In this paper we presented and solved the problem analogous to the the control of a flight simulator: the control of a “locomotion simulator,” or Virtual Motion System. We drastically simplified the complex dynamics in several ways. The intermittent dynamics of legged systems were replaced by continuous cart dynamics, controlled by a simple proportional velocity controller responding to a commanded step input. In the two approaches presented we first include the VMS dynamics as a second order system, and subsequently ignore its dynamic model as well. Of course, we argue that we have still retained the essence of the problem, and show that the optimal control approaches provide satisfactory solutions in the presence of mutually conflicting constraints.

The final test of the relevance of the results presented is experimental validation, which we plan to do in the near future. We are currently working with a one-legged running robot and are going to build a two legged version as well. Both will operate on a high performance treadmill. This system requires a treadmill controller subject to the specifications outlined in Section 3.

There are several fundamental modeling, mechanical design and control challenges to be solved in the future. In the realm of modeling, we feel that experiments need to be performed first to validate our modeling assumptions and to guide further research. This is especially true for the case of human locomotion, where our assumption of a “human proportional velocity controller” might appear overly crude. Independent of this, mechanical designs for computer controlled VMS devices that provide unrestricted planar motion is an unsolved challenge. This is the inverse problem to the omni-directional tracked vehicle. In the realm of control, we need to generalize the simple velocity setpoint control of

the robot/human on the VMS to a continuous reference input where only bounds on the acceleration are known. This situation is more realistic for applications in artificial reality where a human on the VMS is tracking some à priori unknown reference trajectory.

Acknowledgements

This work has been supported in part by an FCAR New Researcher Grant held by the second author. Support for MMM was provided by the Ministry of Culture and Higher Education of Iran. Personal support for MB was provided by the NSERC/Canadian Institute for Advanced Research (CIAR) Industrial Chair in Robotics.

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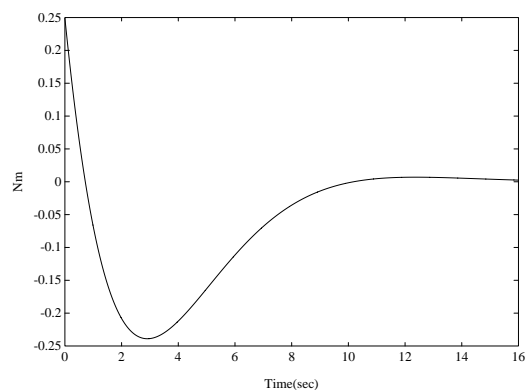


Figure 2: Optimum Control Input, τ

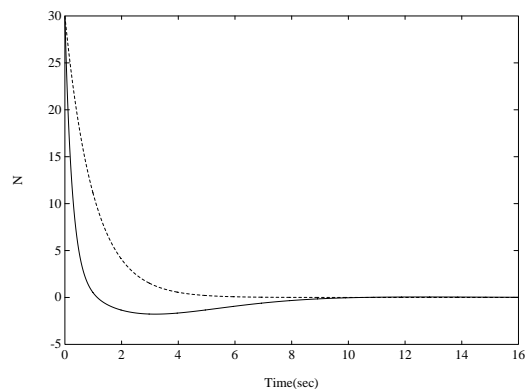


Figure 3: Contact Force (solid = on the VMS, f_c , dashed = on the Ground, f_g)

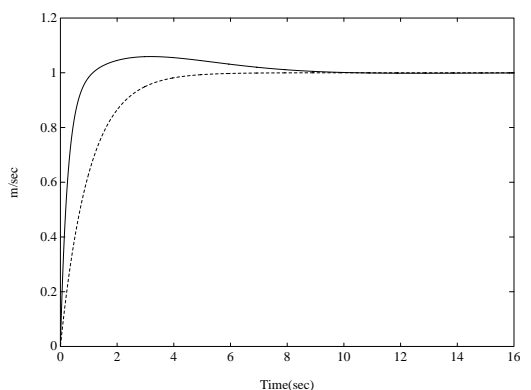


Figure 4: Cart Velocity (solid = relative to VMS, v_{cb} , dashed = ground ref., v_g)

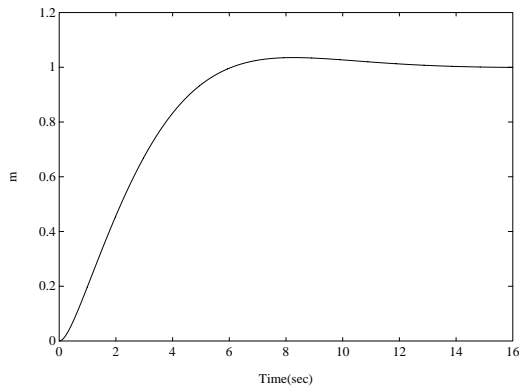


Figure 5: Cart Position, x_c

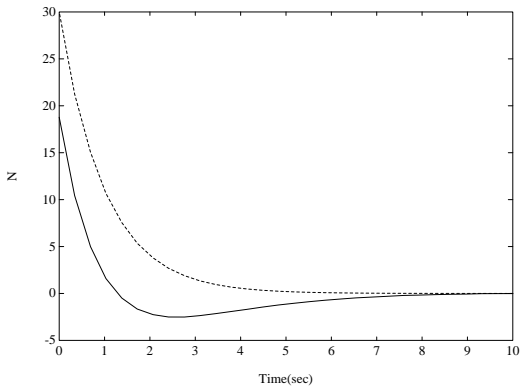


Figure 6: Contact Force (solid = on the VMS, f_c , dashed = on the Ground, f_g)

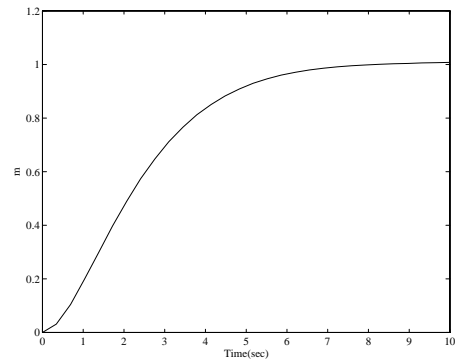


Figure 8: Cart Position, x_c

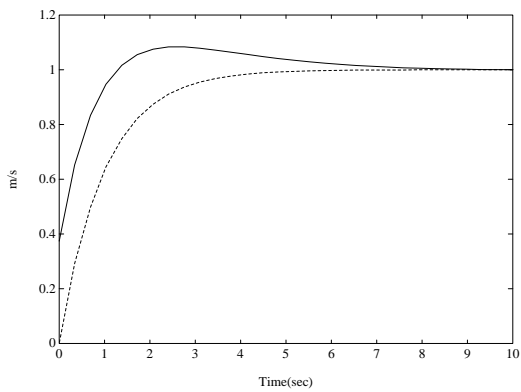


Figure 7: Cart Velocity (solid = relative to VMS, v_{cb} , dashed = ground ref., v_g)