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Analysis of a Simplified Hopping Robot

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Abstract

We offer some preliminary analytical results concerning simplified models of Raibert's hopper. The motivation for this work is the hope that it will facilitate the development of general design principles for "dynamically dexterous" robots.

1 Introduction

This paper presents a preliminary analysis of the limiting behavior of a "hopping ball" controlled by simple sensory feedback to achieve a stable periodic motion in the earth's gravitational field. We take as inspiration and as point of departure, the pioneering work of Marc Raibert whose successful implementation of simple yet appropriate control procedures has resulted in working physical prototypes of stable hopping, running, and cantering gaits [4]. The most striking feature of these control algorithms is their minimal dependence on "higher level intelligence" and elegant reliance upon the intrinsic physical characteristics of actuators and masses. An understanding of the capabilities and limits of such approaches to robot task specification and control seems essential to the reliable construction of "dynamically dexterous robots" in general. This last phrase we understand to mean the problem of robotic interaction with incompletely actuated environments (i.e., the absence of a continuous control input at every mechanical degree of freedom) whose dynamical structure changes in response to the robot's actions. It is our hope that a formal accounting for the success of a particular strategy may suggest control procedures and even design parameter values which generalize to other problems of robotic "dynamical dexterity."

Our present contribution may be summarized as follows. Studying the changes in an energy-like quantity (whose form depends upon the details of the dynamical model selected) brought about by collisions between the robot and environment leads to a discrete dynamical system. The limiting properties of the descrete dynamics describe the salient characteristics of the robot's performance, and may be compared with the true physical setup. This paper concentrates upon the global stability of a unique periodic orbit — a formalization of our intuitive sense of what would constitute a successful hopping strategy. Simulations are then used to validate the particular models investigated.

2 Modeling

Our abstraction of the vertical hopper consists of a body of unit mass in the gravitational field and a leg of zero mass subject to viscous friction, γ. The leg is "actuated" by a pneumatic cylinder which simultaneously acts as an energy storage mechanism. While the pneumatic cylinder gives rise to an inverse spring law $\varphi_{nl}(\chi) \sim 1/\chi$, we will also examine a linear spring $\varphi_l(\chi) \sim \chi$. The actuator is "controlled" by an adjustable "spring constant" $\eta(\chi, \dot{\chi}, t)$ which multiplies the spring law, φ. $\eta(\chi, \dot{\chi}, t)$ implements as closely as possible the feedback strategy as described in Raibert's book. He divides the time of one complete vertical hop into compression, thrust, decompression and flight phase which begin at touchdown (td), bottom (b), end-of-thrust (et) and liftoff (lo), respectively.

For the nonlinear[linear] robot, $\varphi_{nl[l]}$, the effective feedback control law may be specified as

$$\eta_{nl[l]}(\chi, \dot{\chi}, t) = \begin{cases} \eta_1 & \dot{\chi} < 0, \chi < \chi_{td} \\ \tau\chi \quad [\frac{\tau}{\chi_0 - \chi}] & t \in (t_b, t_{et}) \\ \eta_2 = \tau\chi_{et} \quad [\frac{\tau}{\chi_0 - \chi_{et}}] & \chi_{et} < \chi < \chi_{lo} \\ 0 & \text{otherwise.} \end{cases}$$

We will find it useful to make a conceptual distinction between the "robot" — the nature of the spring mechanism, φ, and its feedback control strategy, η — and the "environment" — the friction in the leg, the force of gravity, the location of ground, etc. The forces exerted by the robot upon the environment and the interaction of the environment with the robot may now be written as $F_r \triangleq \ddot{\chi} - \eta(\chi, \dot{\chi}) \cdot \varphi(\chi)$ and $F_e \triangleq -g - \sigma(\chi, \dot{\chi})\gamma$. $\sigma(\chi, \dot{\chi})$ switches off friction during flight. Coupling the dynamical structure of the environment to the robot, $F_r = F_e$ now gives our model,

$$\ddot{\chi} + \sigma(\chi, \dot{\chi})\gamma\dot{\chi} - \eta(\chi, \dot{\chi}, t)\varphi(\chi) + g = 0. \quad (1)$$

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3 Analytical Results

The chief motive for the simplifications of the true physical set-up which lead to this model is that the resulting continuous dynamical sytem (1) may be integrated in a piecewise fashion. Defining a suitable "Poincaré section" in the continuous phase space thus leads to an explicit discrete "return map" [3]. We may now summarize some preliminary analytical results concerning the limiting behavior of the discrete return map dynamics. For a more detailed explanation of the equations as well as the proofs of the following Propositions, see reference [2].

First, we offer the results of an analysis of the linear spring version of (1) in the complete environment, F_e. A "simplified" version of the control law, η_l, where the spring constant is unchanged before and after the thrust phase, and the touchdown and liftoff points lie on the same axis, leads to the following discrete dynamics,

$$\chi_{j+1} = \left([\psi_t - \sqrt{\chi_j}]^2 + \dot{\psi}_t^2\right) \exp\left\{-\kappa\left[\pi - \arctan\left(\frac{\dot{\psi}_t}{\sqrt{\chi_j} - \psi_t}\right)\right]\right\}. \tag{2}$$

Here, χ_j represents the height of the robot at the bottom point of the "jth" hop. The parameters ψ_t and $\dot{\psi}_t$ represent, respectively, the change in hopper position and velocity in a normalized coordinate frame and are set by the operator. The parameter κ is a system parameter depending on the spring constant and viscous friction, and is not under the operator's control.

Proposition 1 *The dynamical system (2) has a unique, globally attracting fixed point on the domain* $\mathcal{D} \triangleq (0, \infty)$.

This states that the linear-spring based hopper, when subjected to a simplified version of the Raibert control scheme in a fairly accurate version of the true environment, is guaranteed to have a globally attractive stable periodic limiting trajectory. In other words, a stable periodic gait develops from any initial conditions at start up.

Second, we consider a nonlinear spring which is much closer to the physical apparatus described in Raibert's book. In order to retain piecewise closed form integration of the continuous model, we remove the viscous friction forces and gravity during the stance phase, let the time of thrust go to zero and again, assume that touchdown and liftoff points lie on the same axis. The new discrete dynamics are given by

$$\chi_{j+1} = \chi_{lo} \exp\left\{-\frac{\tau\chi_j}{\eta_1} \ln \frac{\chi_{lo}}{\chi_j}\right\}. \tag{3}$$

Again, χ_j represents the height of the robot at successive bottom points and the design parameter χ_{lo} represents the fixed height where both robot liftoff and touchdown occur, as chosen by the operator.

Proposition 2 *The dynamical system (3) has a unique fixed point,* χ^*, *on the domain* $\mathcal{D} \triangleq (0, \chi_{lo})$, *which is locally asymptotically stable if and only if* $\chi^* \in \mathcal{D}_1 \triangleq (\chi_{lo}/e^2, \chi_{lo})$. *If* χ^* *is not a local attractor, i.e.,* $\chi^* \in (0, \chi_{lo}/e^2) \triangleq \mathcal{D}_0$, *then there exists at least one orbit of period two, i.e. a fixed point of* $g \triangleq f \circ f$, *which is not a fixed point of* f.

The latter states that an inverse-spring-law based hopper, when subjected to a closer approximation of the true Raibert control scheme in a trivialized version of the environment, will always have a periodic trajectory, but that this may be unstable, and, if it is, must evince a "doubly looped" trajectory in phase space (i.e. there must be a period-two orbit of the discrete dynamical system). In other words, depending upon choice of the design parameter χ_{lo}, initial conditions at startup time might lead to a doubly periodic "limping" gait. [1]

4 Simulations

Simulations carry a great deal of importance for these investigations. In the absence of physical experiments on the real apparatus, they provide the only verification of the relevance of the simplified model (1) described above. In this section we present a number of simulations which serve to validate the model with respect to the real hopper's trajectories. These plots illustrate the formal results, Propositions 1 and 2, as well.

As a first check on the validity of our model, we compare the simulations of the linear simplified model Figure 3 and of the full nonlinear model Figure 2 with a plot of the physical system lifted straight out of Raibert's book [4] — Figure 1. Starting at top, the vertical hopper goes through touchdown (note the counterclockwise direction) and compression to bottom. Until liftoff, some part of the trajectory constitutes the thrust phase, which is not clearly distinguishable in this plot. After liftoff, the hopper completes the cycle at the top. The same sequence of events attaches to our simulation plots — Figures 2 through 4 — with the exception that they evolve in a clockwise fashion. Our figures also depict transient trajectories: a dashed trajectory leaves from initial conditions outside the closed curve and a solid line trajectory leaves from inside at the solid dot. These three plots exhibit significant similarity. Figure 4 shows the stable period two orbit of the full nonlinear model as predicted by Proposition 2 using the simplified nonlinear model. If we restrict ourselves to the stable period one orbits, we can see, that both the linear and the nonlinear models reveal similar qualitative behavior which is maintained for a large range of parameter settings.

[1] This term was coined by Raibert in a personal communication.

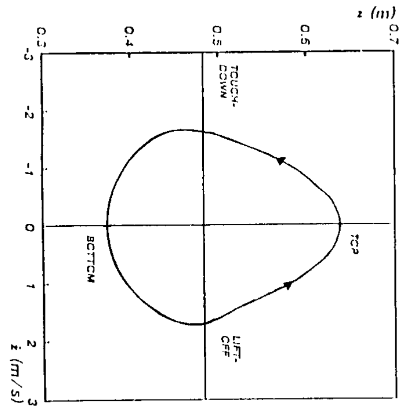

Figure 1: Raibert's hopper

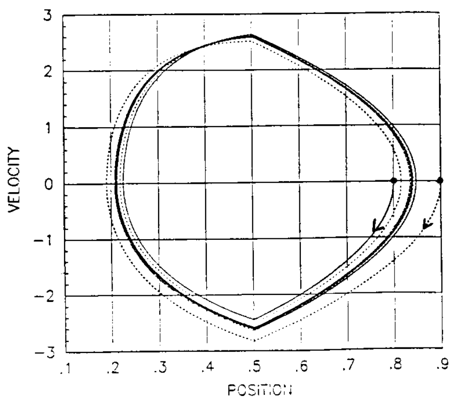

Figure 2: Stable Period 1, full nonlin. model

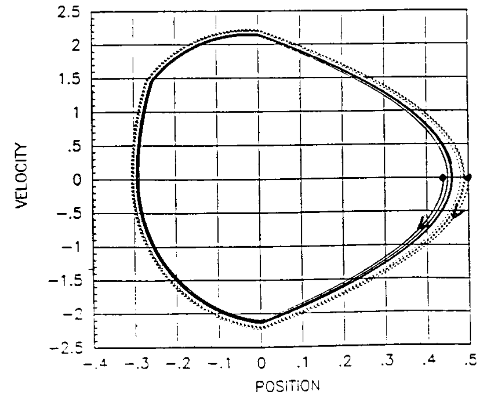

Figure 3: Simplified, linear model

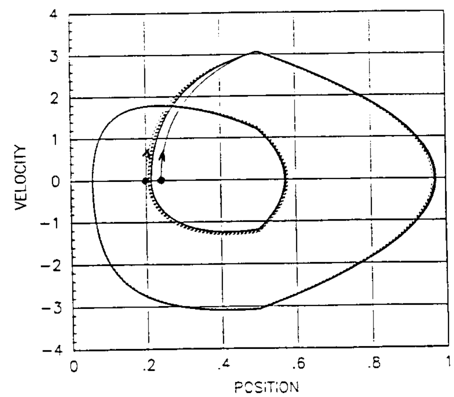

Figure 4: Stable Period 2, full nonlin. model

5 Conclusion

We construct a simplified model of a "dynamically dexterous" robot, Raibert's Hopper, and investigate the effect of his elegant, physically based control strategies. Analysis of induced discrete dynamics leads to strong conclusions concerning global limiting properties. These conclusions are then verified by computer simulation of the simplified models, whose correspondence to the true physical apparatus is seen to be acceptable as well.

This analytical approach seems to enjoy a more fundamental generality than apparent in the present paper. For example, in the design of control algorithms for another member of this class of "dynamically dexterous robots" — a simple prototype juggling robot — a very similar point of view appears to result in successful performance [1]. These insights suggest the existence of a larger unified body of general analytical tools and control algorithm design principles for this class of robotic tasks that await further exploration.

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